

# The de Bruijn graph and genome assembly



Lecture slides adapted from the dBG lecture slides of Ben Langmead.  
All slides in this lecture marked with “\*” courtesy of Ben Langmead.

# Different kind of graph

“tomorrow and tomorrow and tomorrow”

# Different kind of graph

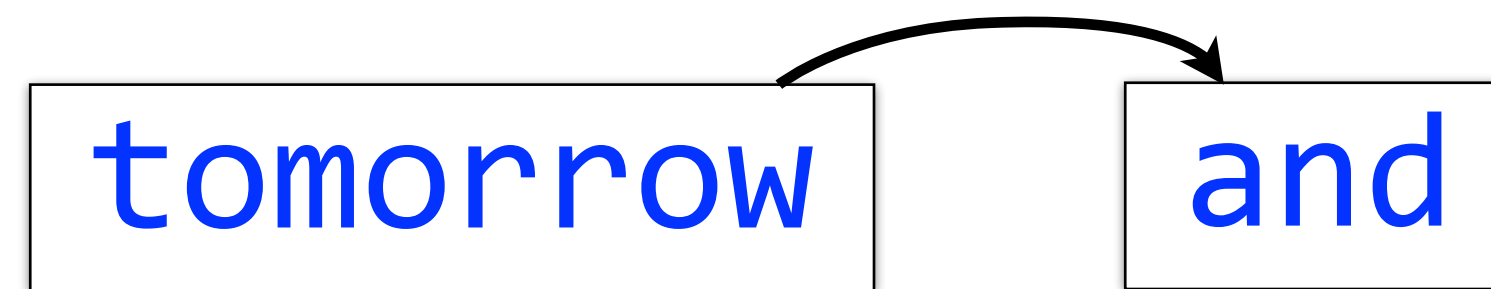
“tomorrow and tomorrow and tomorrow”

tomorrow

and

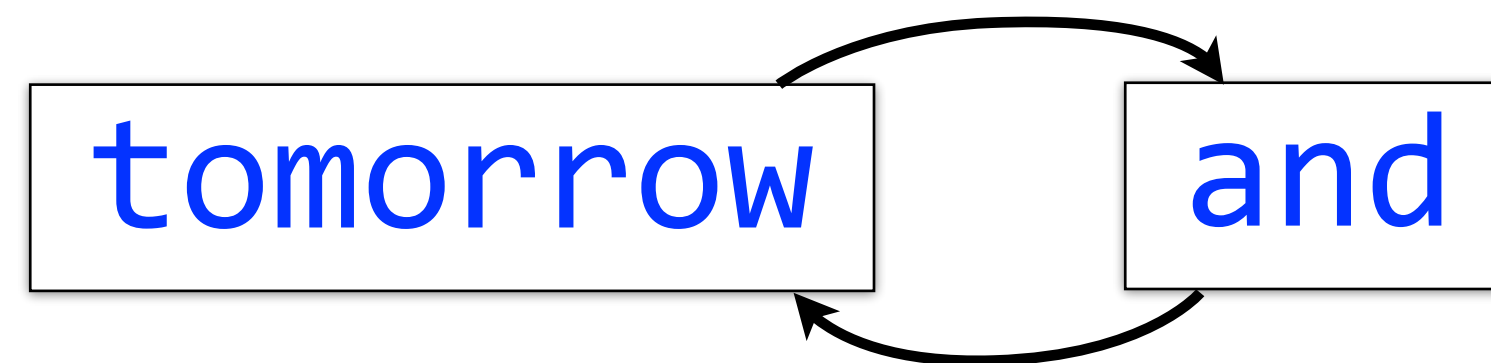
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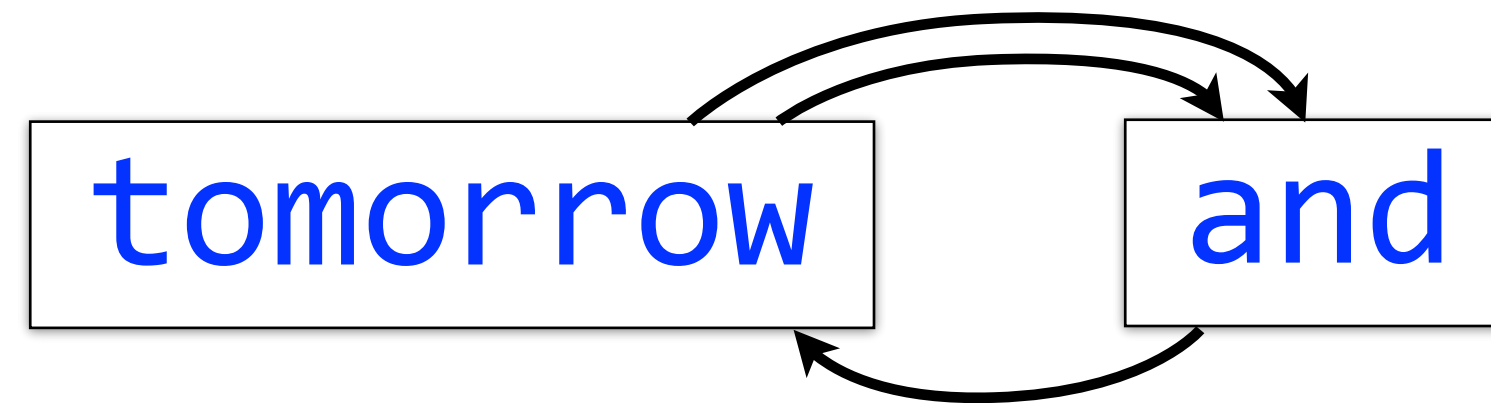
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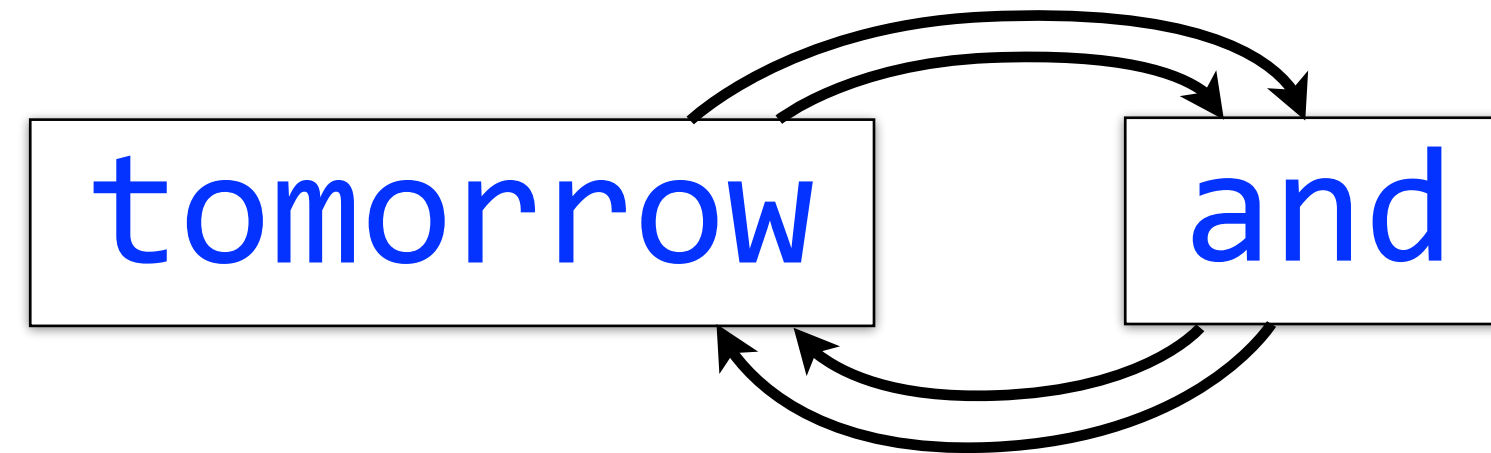
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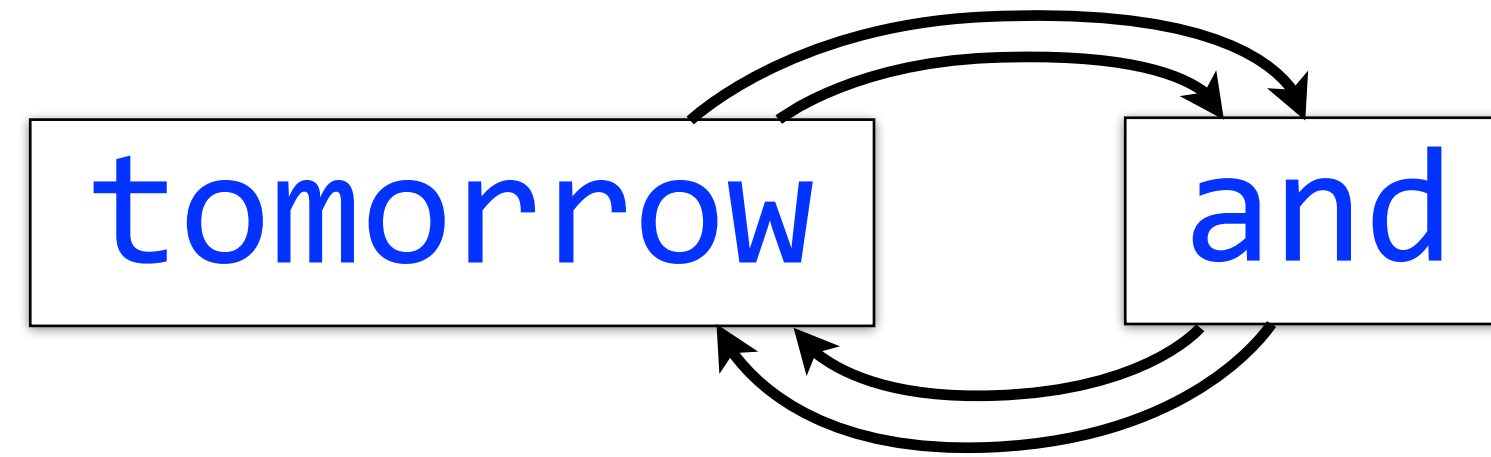
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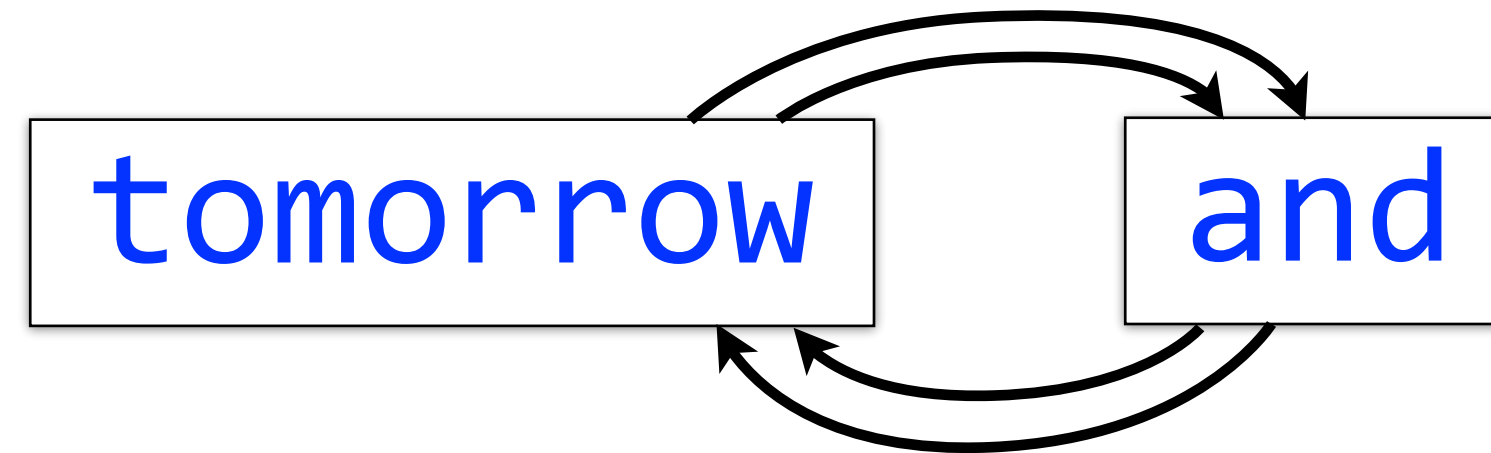
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An edge represents an ordered pair of adjacent words in the input

# Different kind of graph

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An edge represents an ordered pair of adjacent words in the input

Multigraph: there can be more than one edge from node A to node B

# De Bruijn graph

genome: **AAABBBBA**

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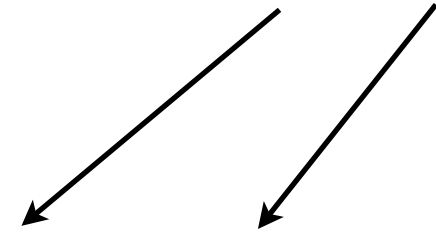
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L/R 2-mers: **AA AA**



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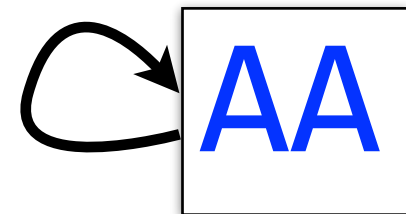
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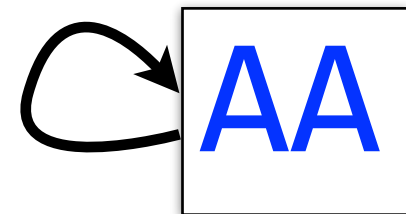
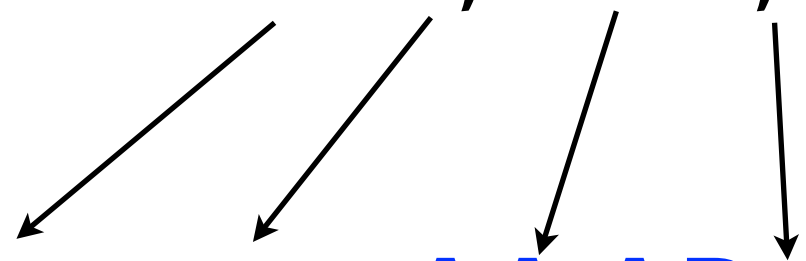


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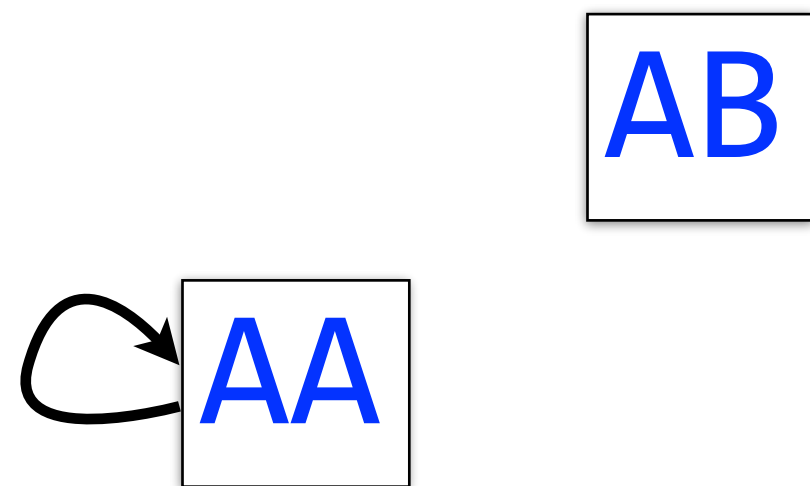


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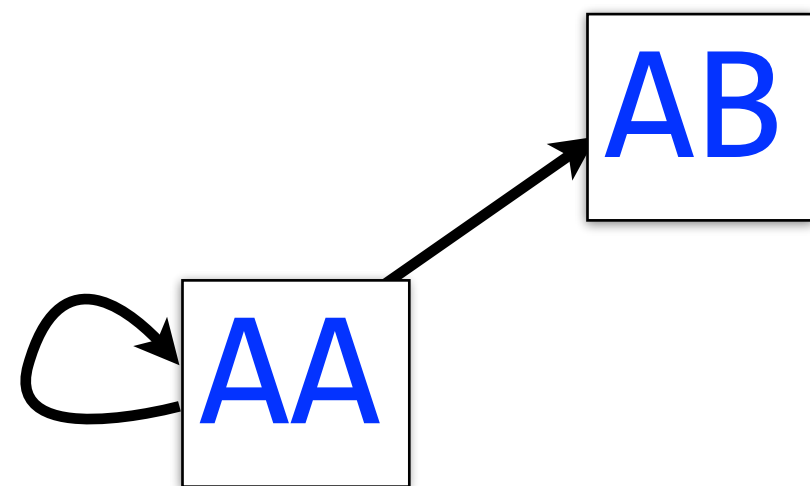


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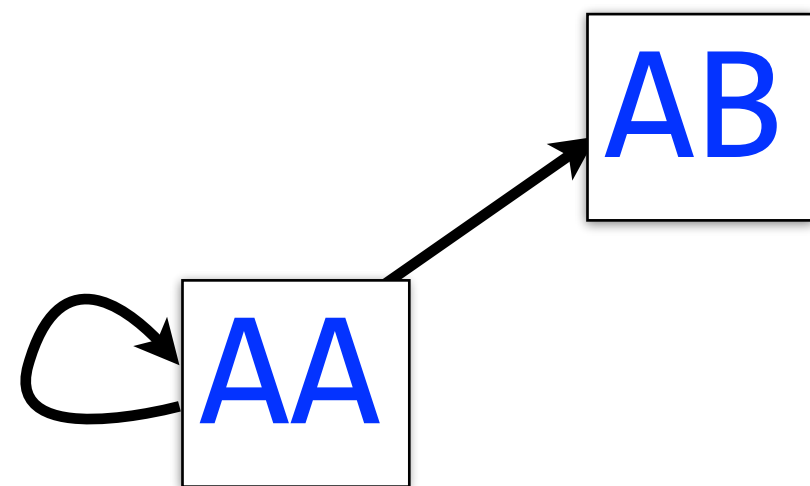


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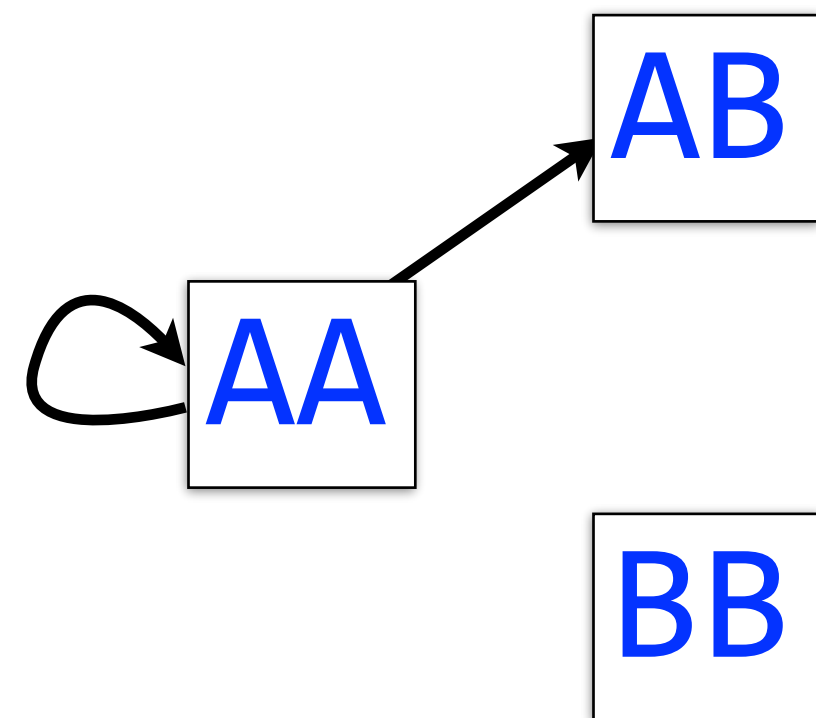


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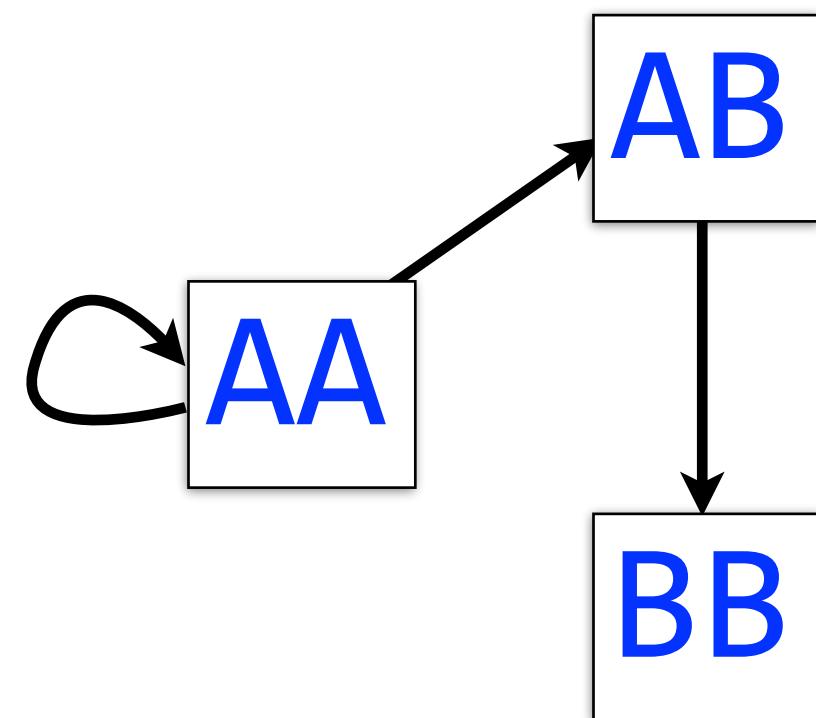


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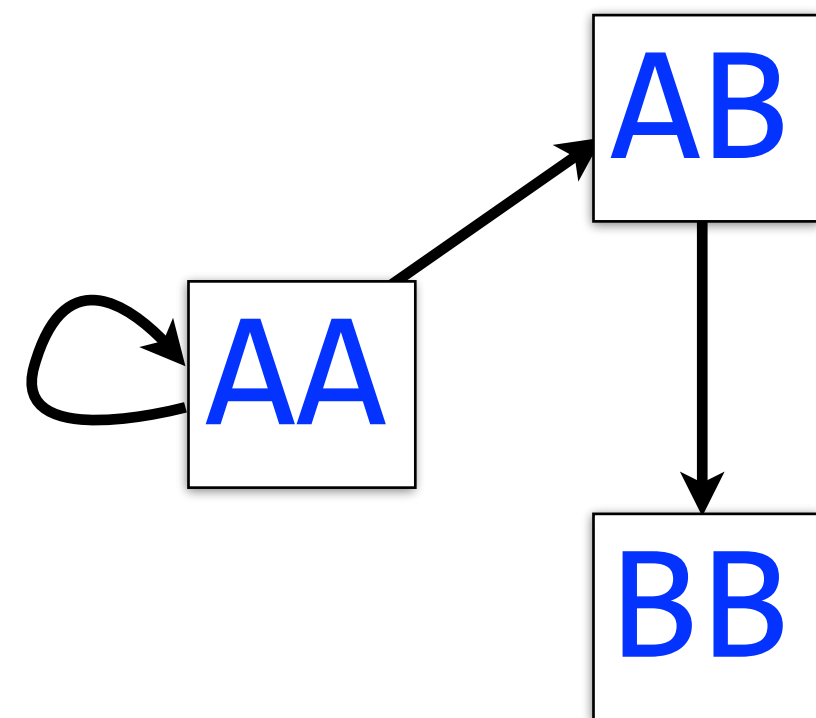


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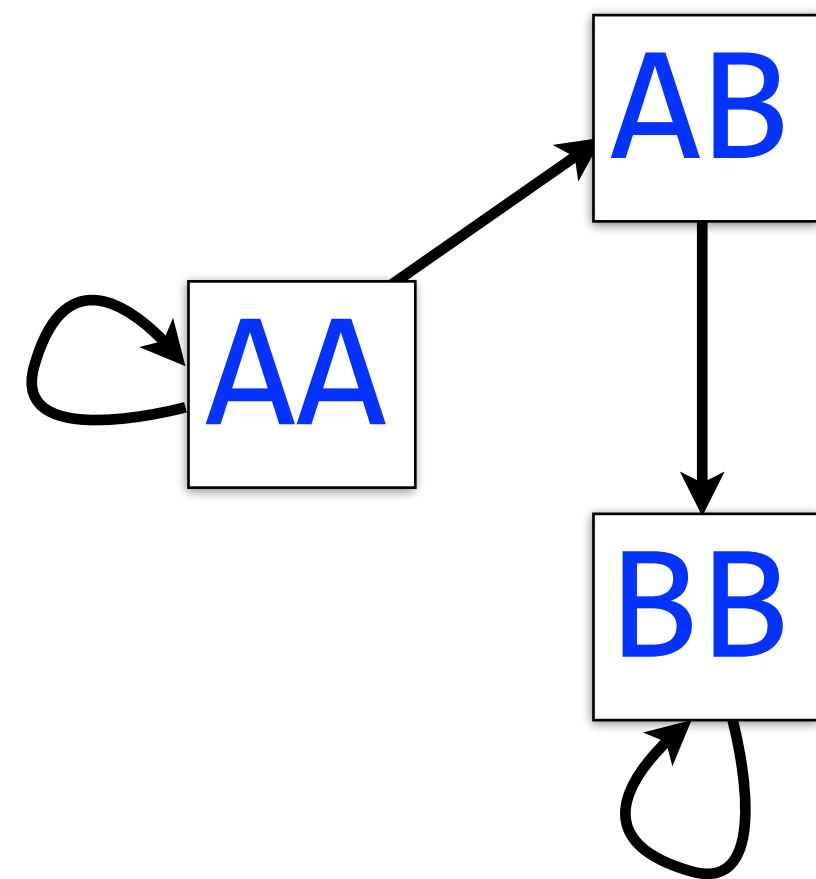


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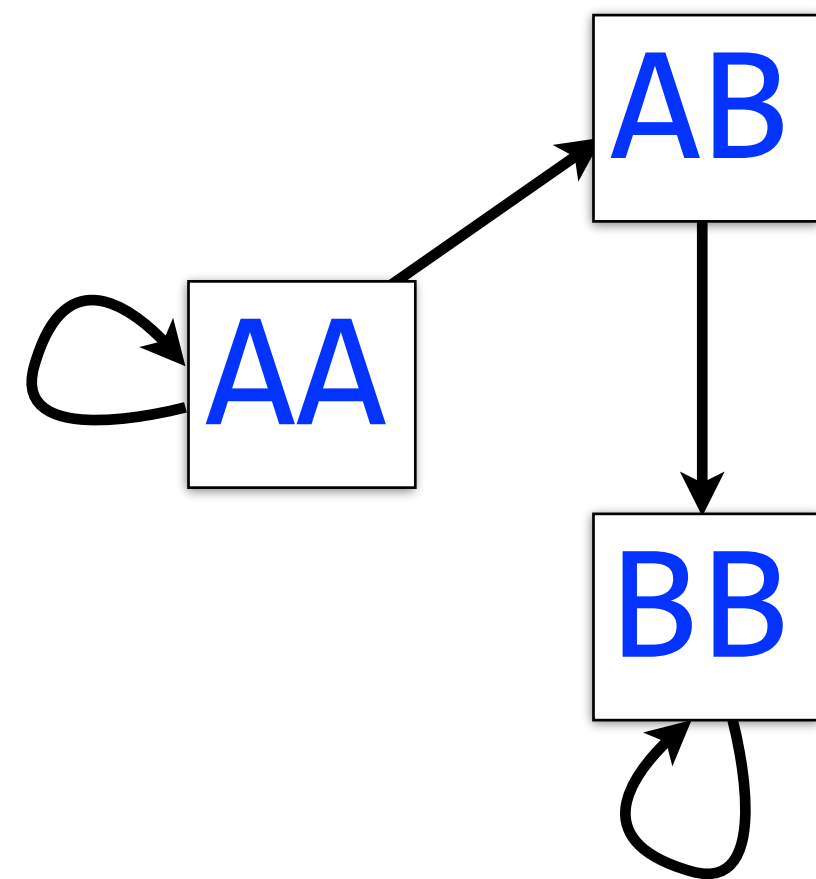


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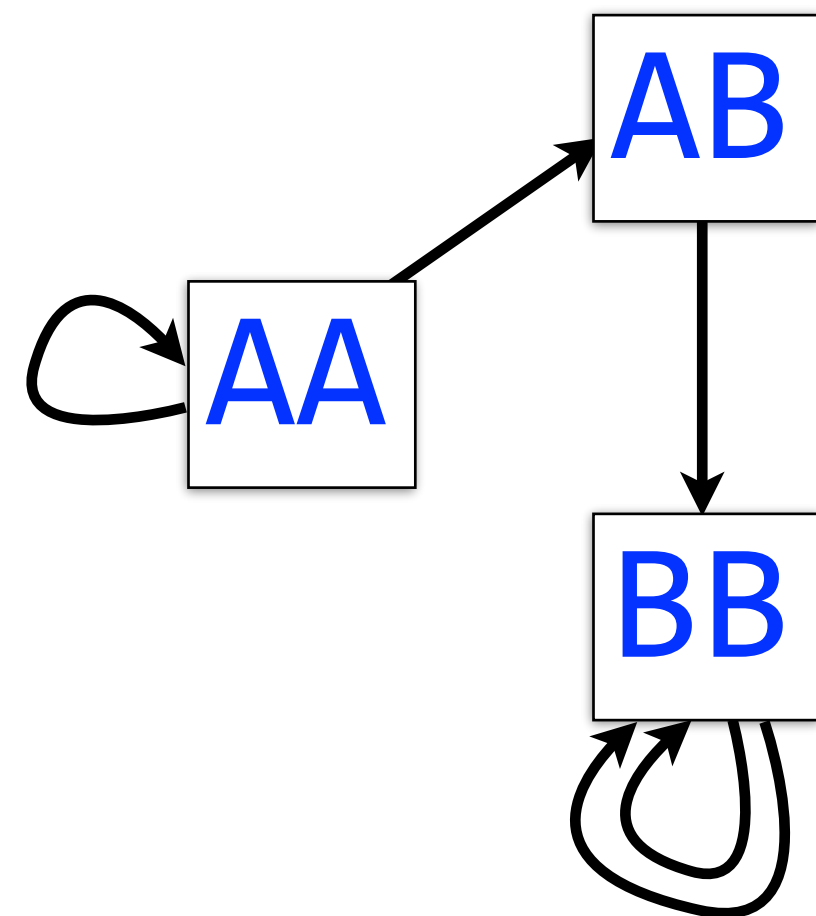


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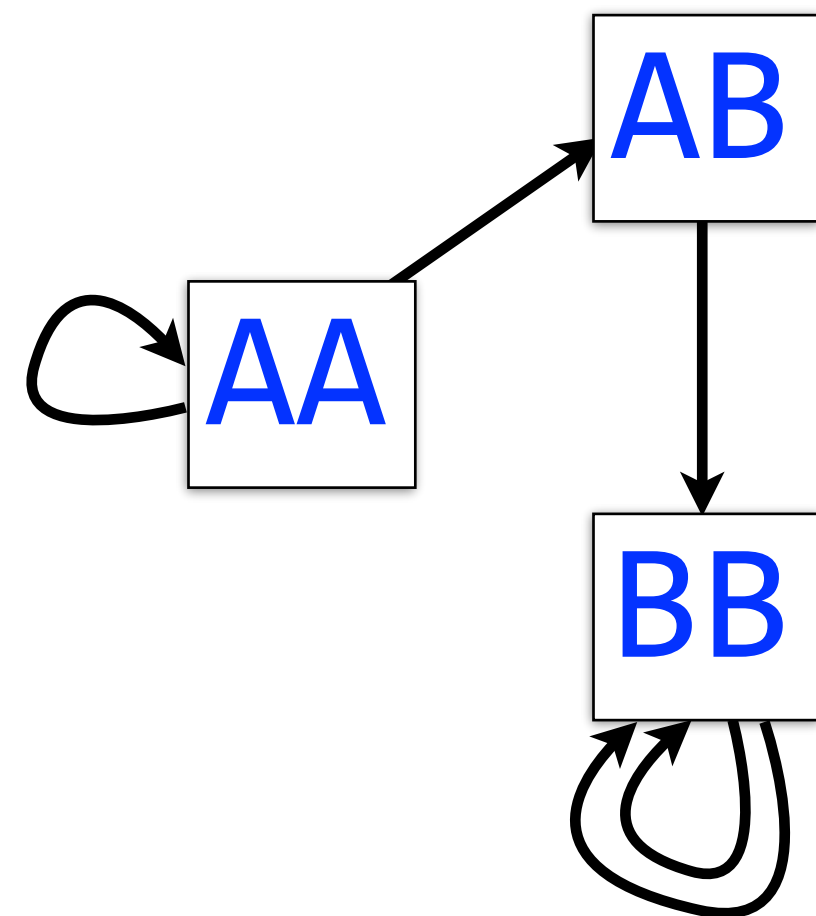


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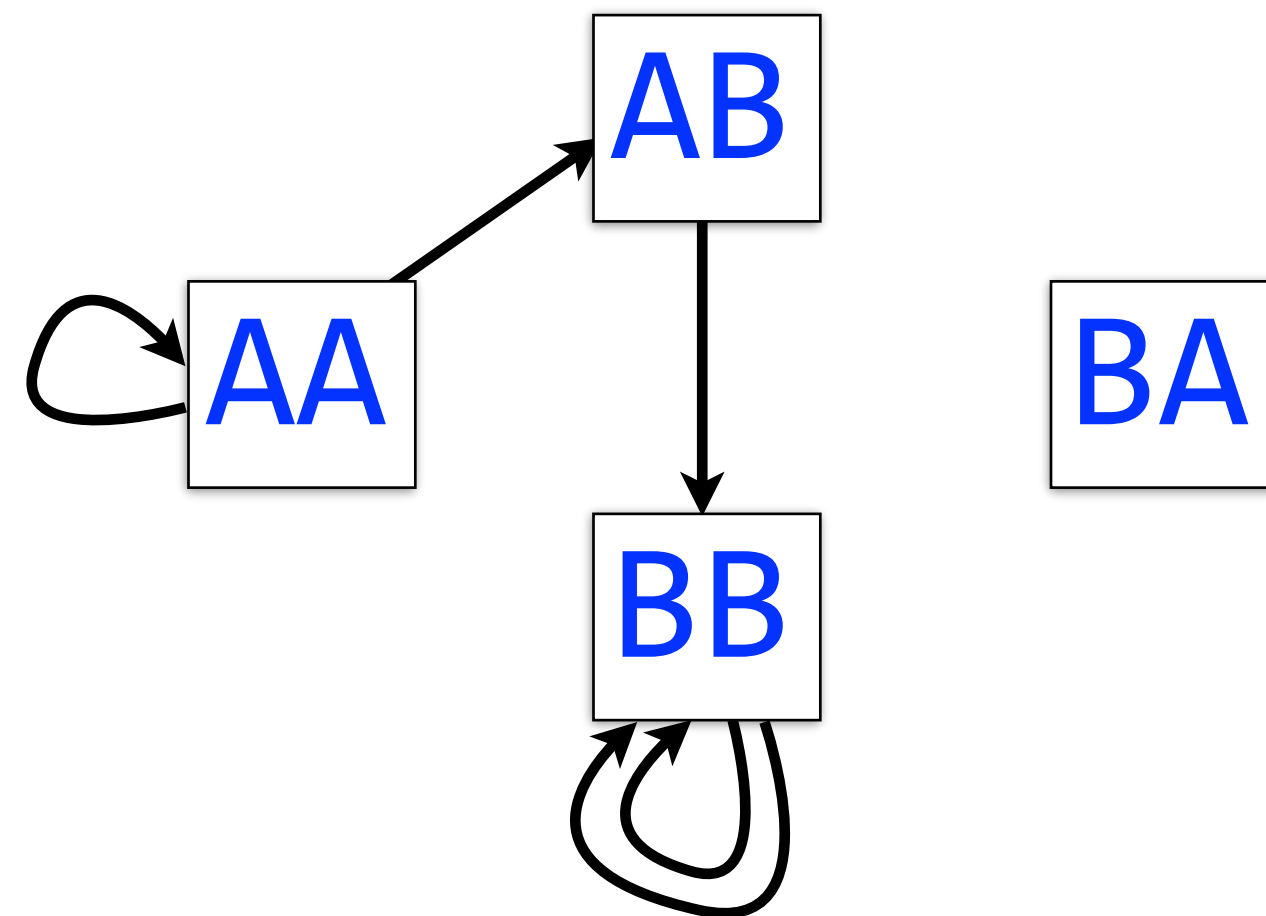


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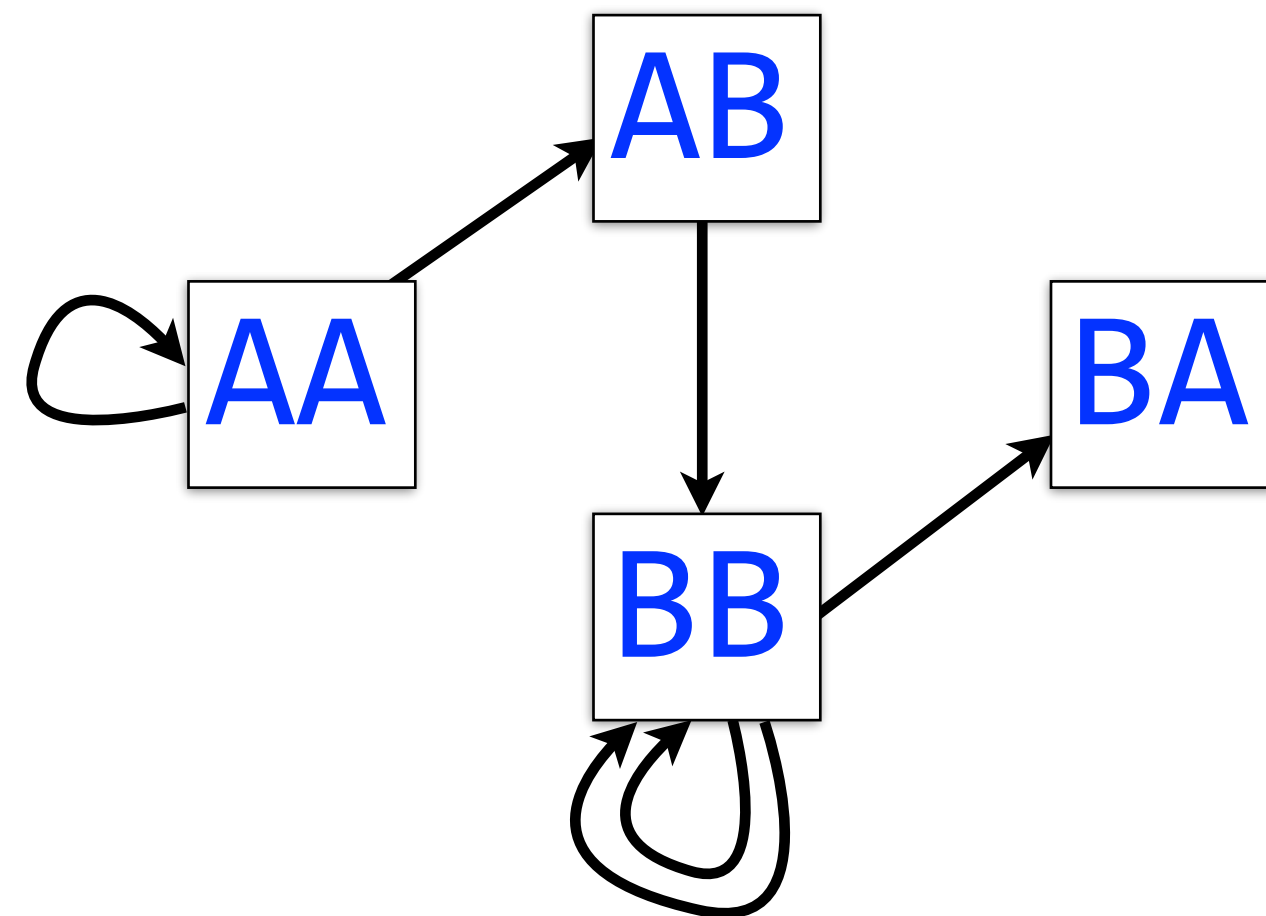


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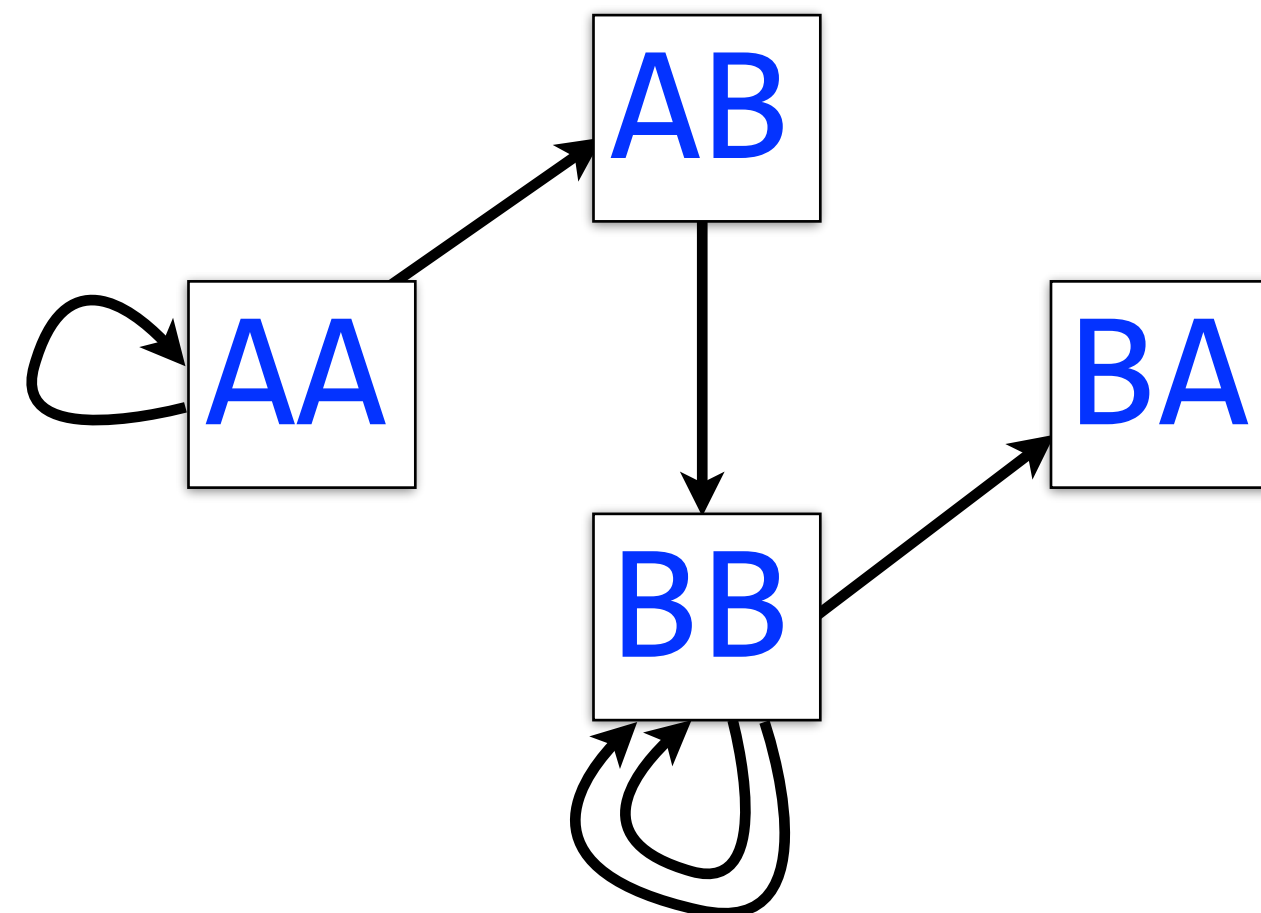


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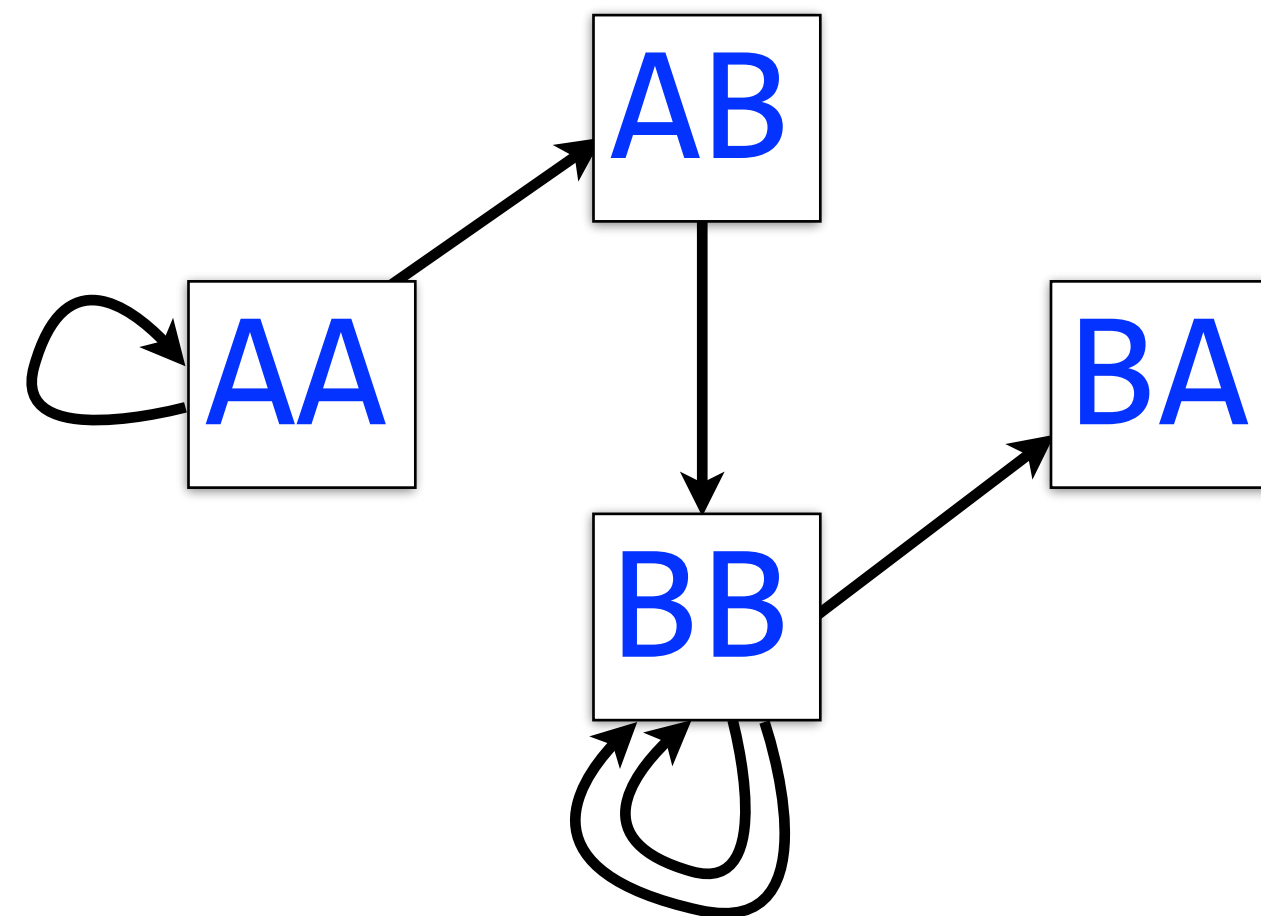
One edge per k-mer

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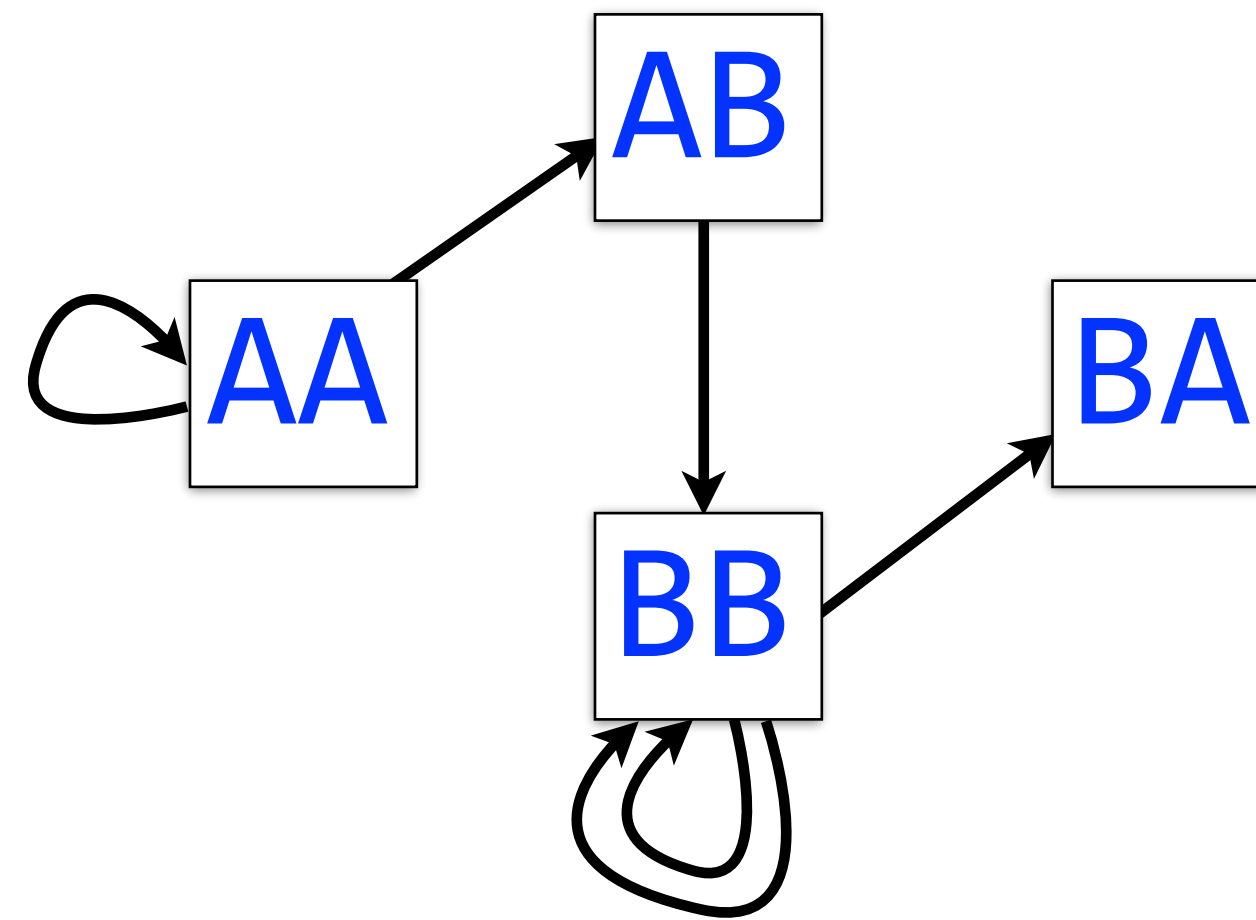
L/R 2-mers: **AA AA AA AB AB BB BB BB BB BA**



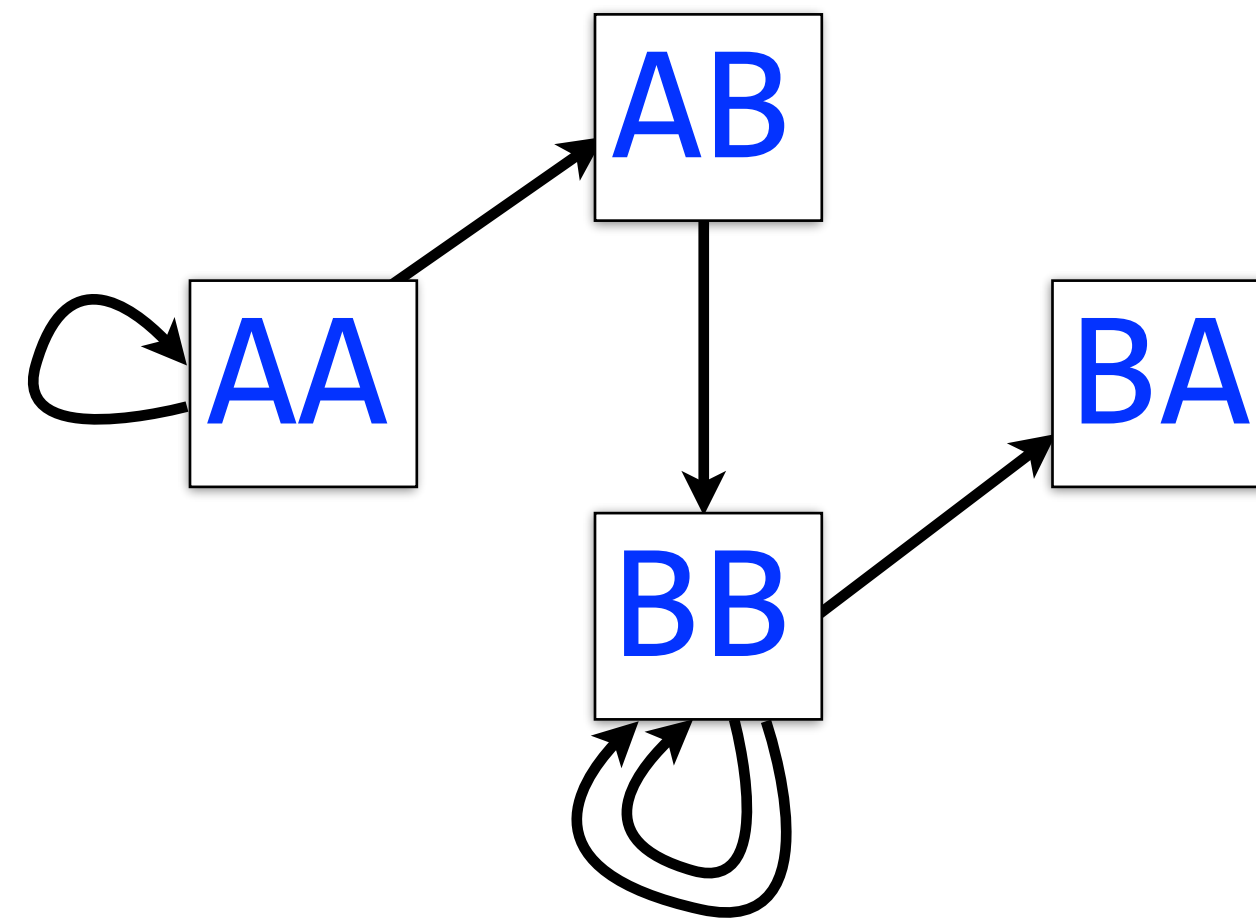
One edge per k-mer

One node per distinct k-1-mer

# De Bruijn graph

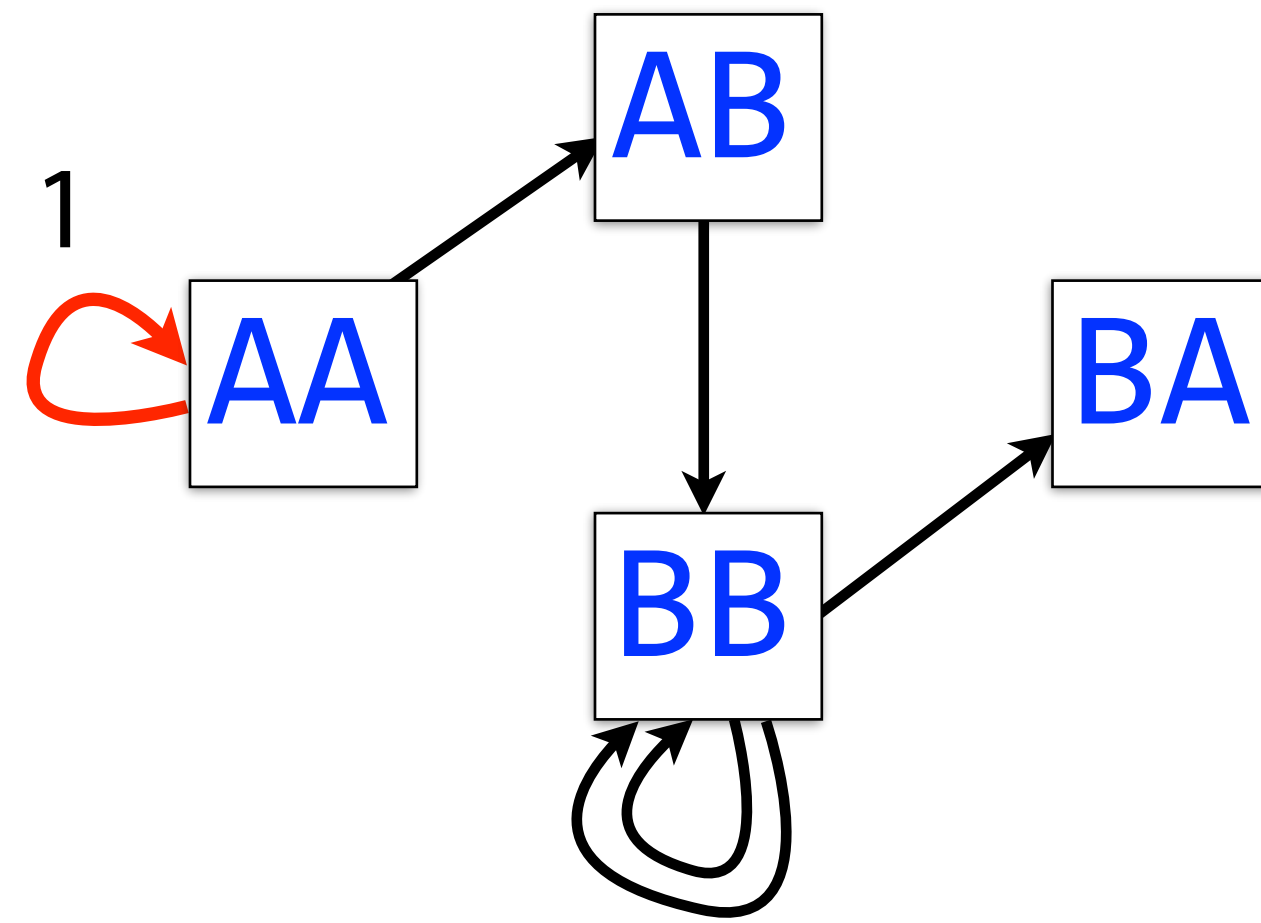


# De Bruijn graph



Walk crossing each edge exactly once gives a reconstruction of the genome

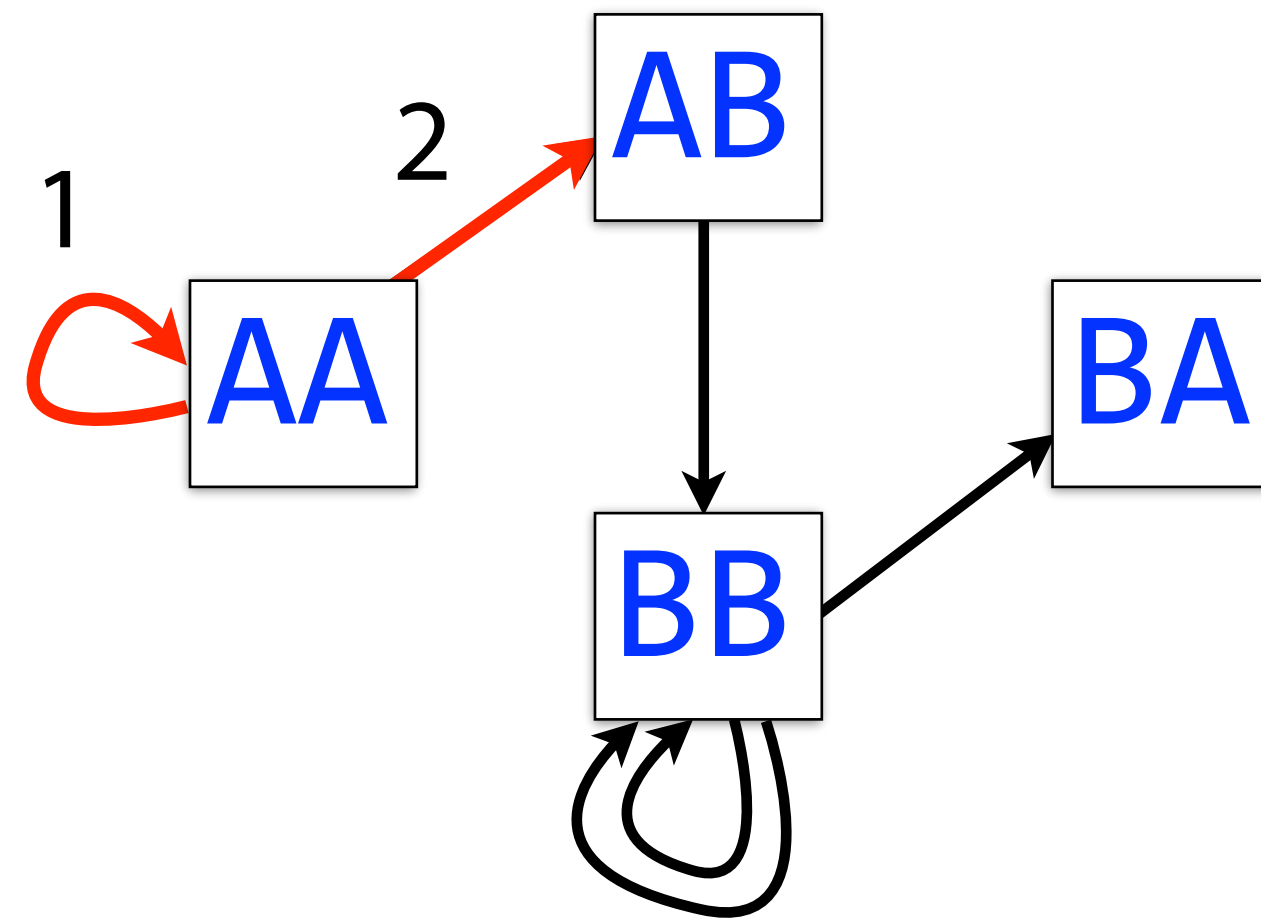
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AAA

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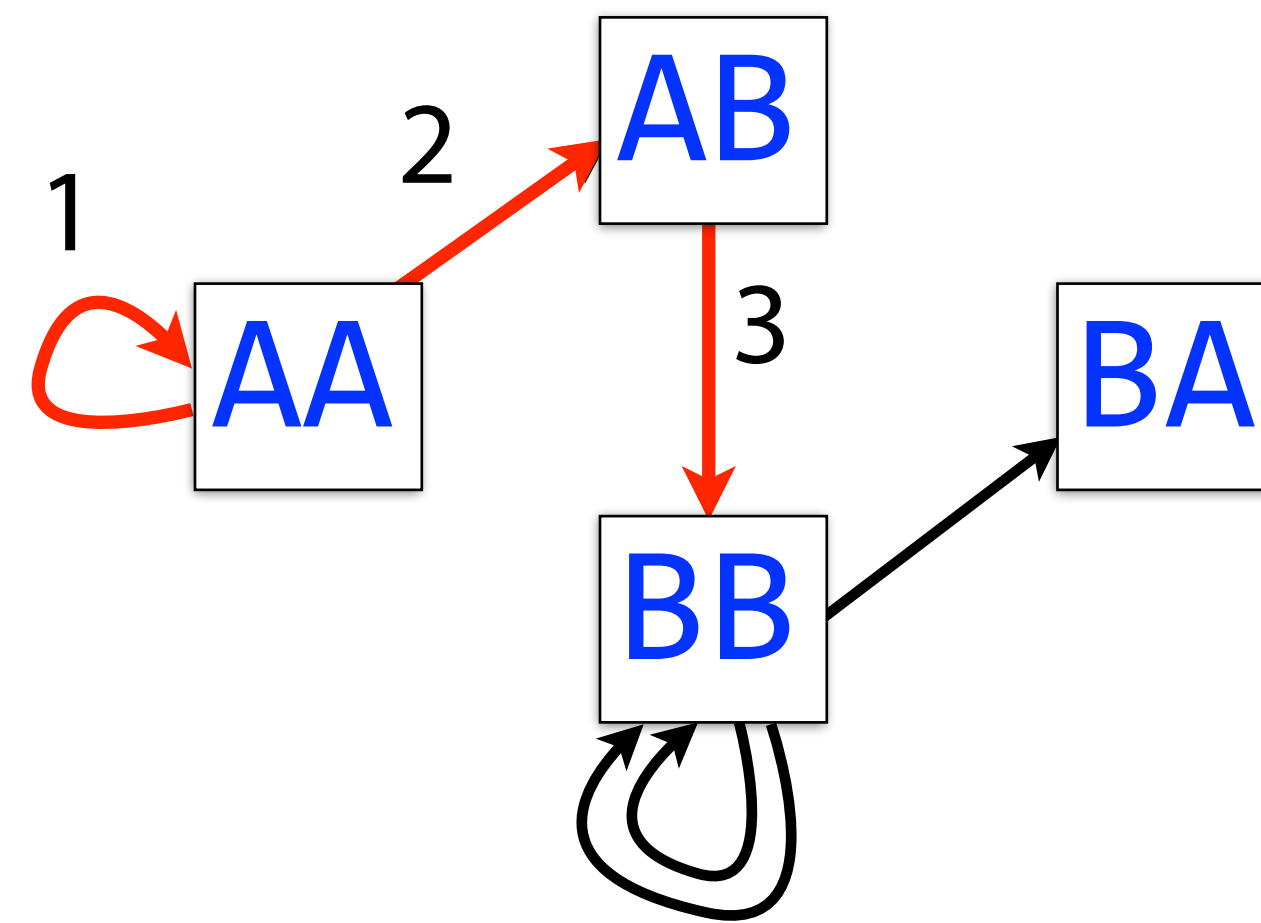
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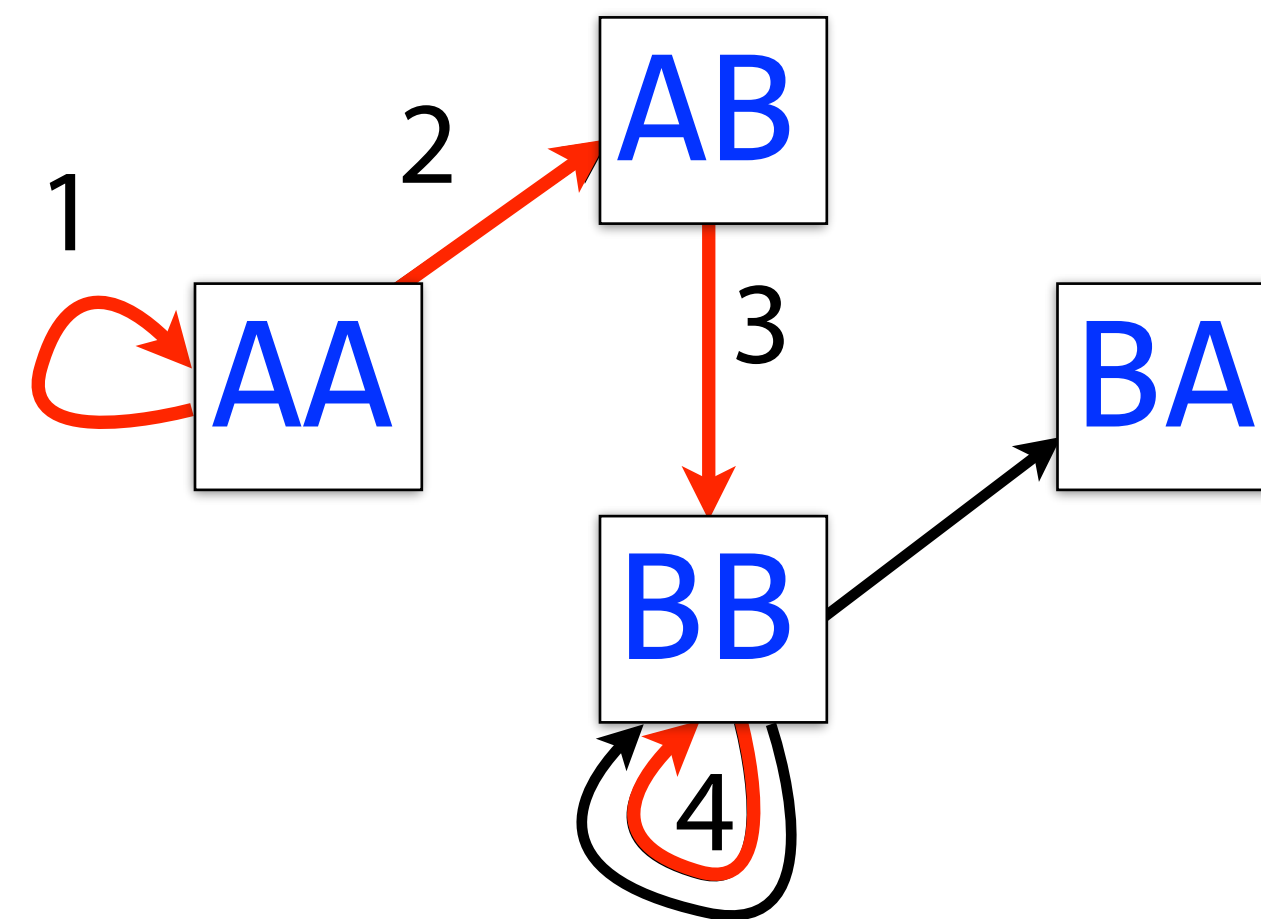
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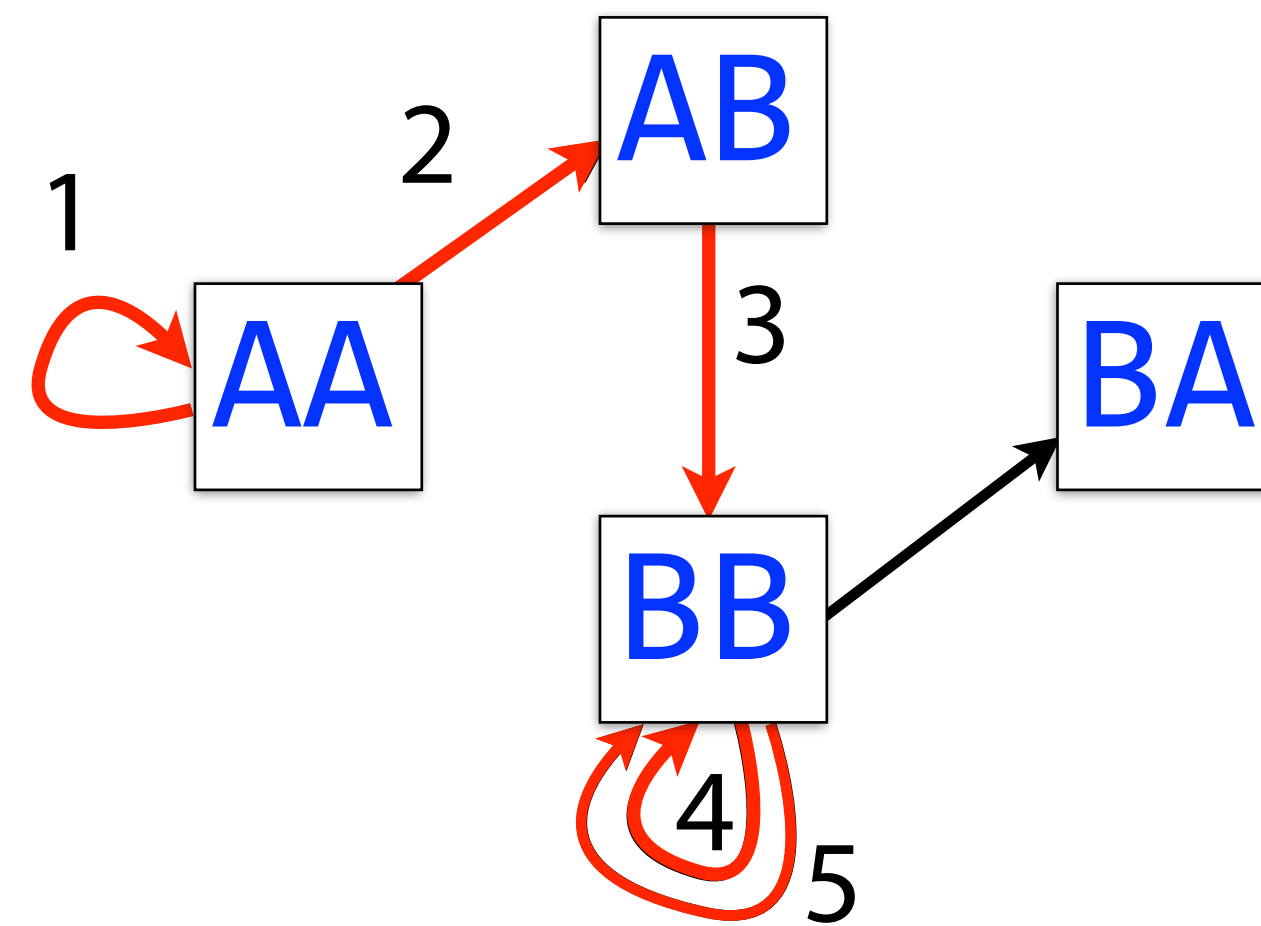
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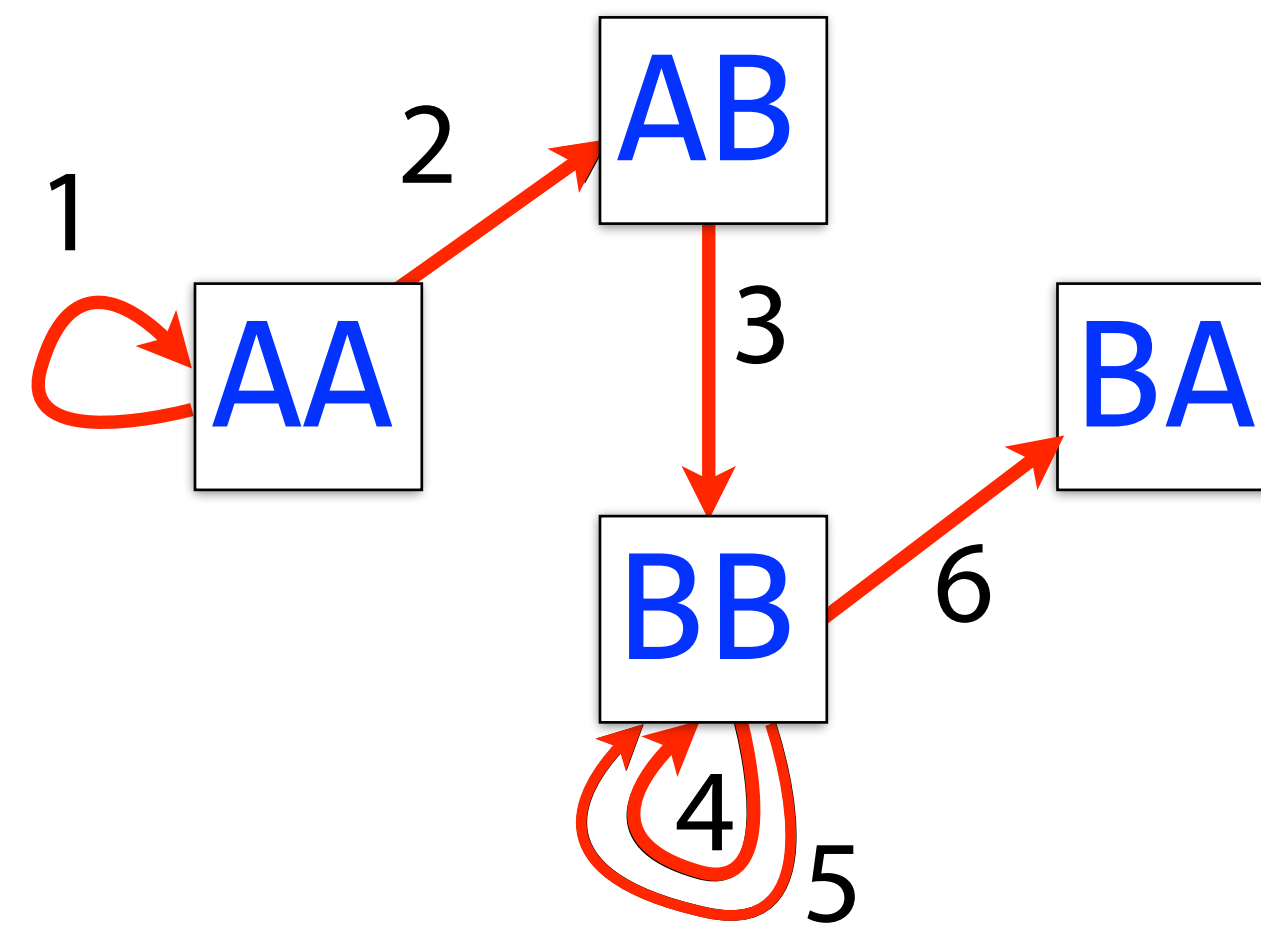
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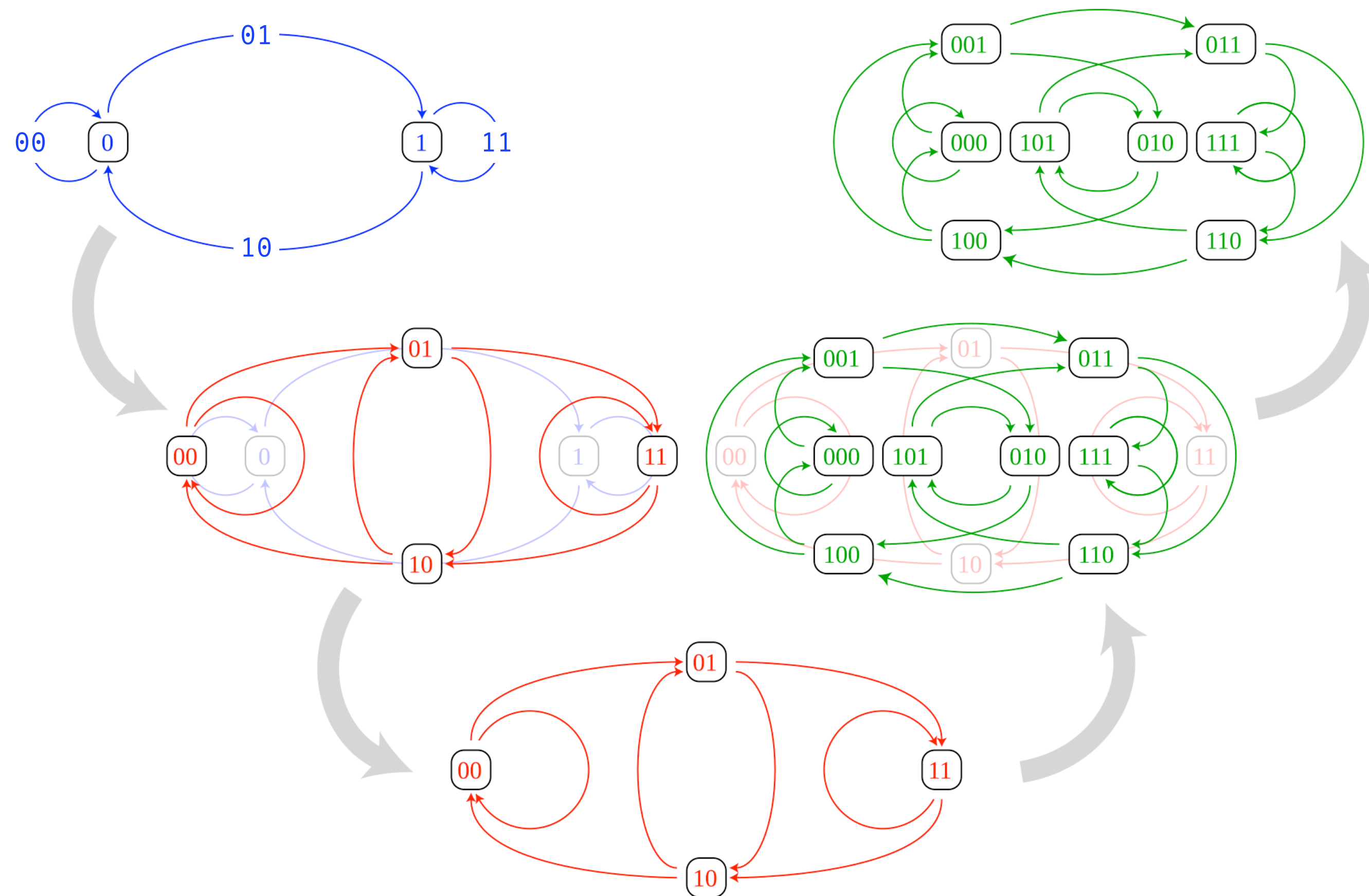
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# The De Bruijn graph (DBG, dbG) & its variants

## Origins

### DBG on binary alphabet



Directed graph representing overlaps between sequences of symbols.

### Vertices:

$$V = S^n = \{(s_1, \dots, s_1, s_1), (s_1, \dots, s_1, s_2), \dots, (s_m, \dots, s_m, s_m)\}$$

$n$  is the “order” of the DBG

### Edges:

$$E = \{((t_1, t_2, \dots, t_n), (t_2, \dots, t_n, s_j)) : t_i \in S, 1 \leq i \leq n, 1 \leq j \leq m\}$$



Nicolaas Govert de Bruijn

(“de brun” or “de brown”;  
not “de bru-jin”)

# The De Bruijn graph (DBG, dbG) & its variants

Popularized for genome assembly (Note: induced subgraph of *actual* DBG)

## An Eulerian path approach to DNA fragment assembly

Pavel A. Pevzner\*, Haixu Tang<sup>†</sup>, and Michael S. Waterman<sup>†‡§</sup>

\*Department of Computer Science and Engineering, University of California, San Diego, La Jolla, CA; and Departments of <sup>†</sup>Mathematics and <sup>‡</sup>Biological Sciences, University of Southern California, Los Angeles, CA

Contributed by Michael S. Waterman, June 7, 2001

Original Genome:

ATCGATCGTA

Sequencing Reads:

ATCG

TCGA

CGAT

GATC

ATCG

TCGT

CGTA

Unique k-mers (k=3)

ATC, CGT, CGA, TCG, GAT, GTA

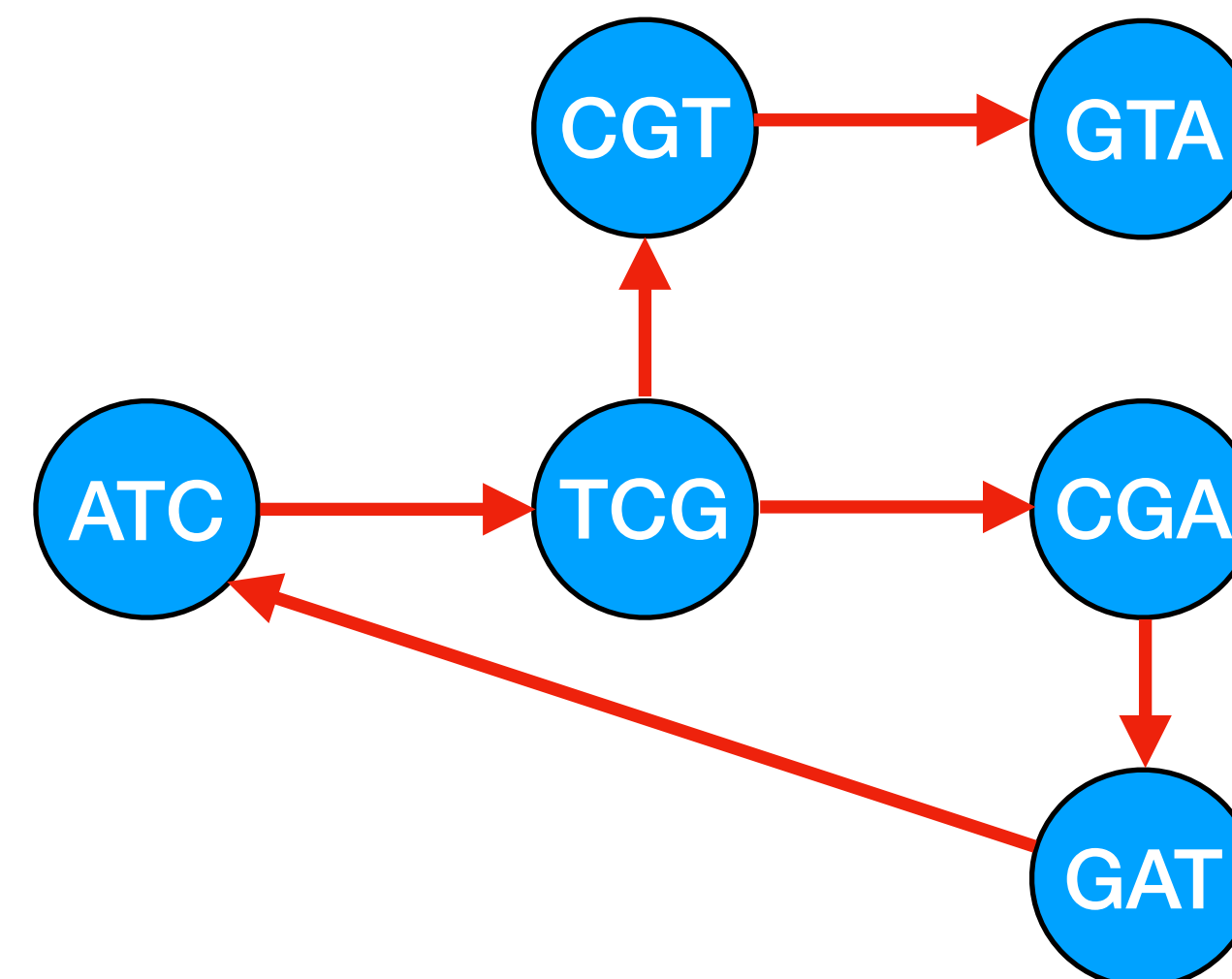
Eulerian Path:

ATC→TCG→CGA→GAT→ATC→TCG→CGT→GTA

Reconstruction:

ATC+G+A+T+C+G+T+A = ATCGATCGTA

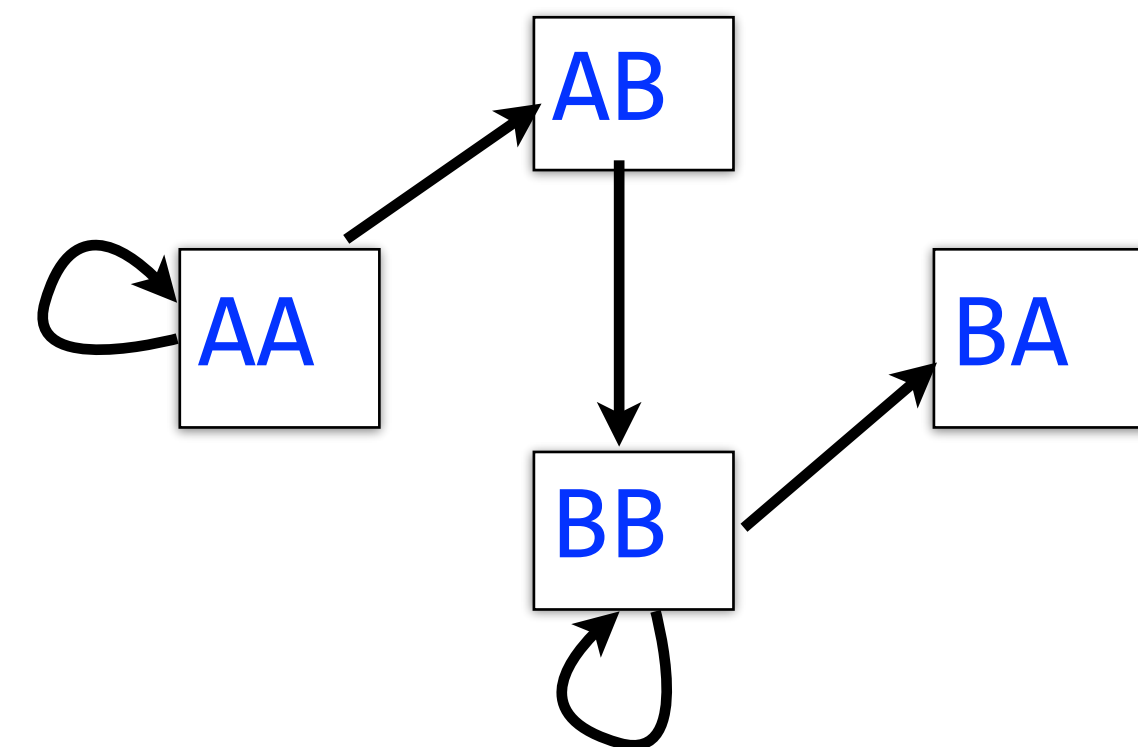
De Bruijn Graph:



- Popularized as sequencing throughput increased.
- Vertices are (k-1)-mers, edges are k-mers
- Replaced expensive inexact read “overlap alignment” with exact matching 2 (k-1)-mers are joined by an edge if the co-occur in the same k-mer
- Vastly improved scalability (time & memory)
- Beautiful theory (in the “perfect” case)
- Works well in practice (with appropriate heuristics)

# Eulerian walk definitions and statements

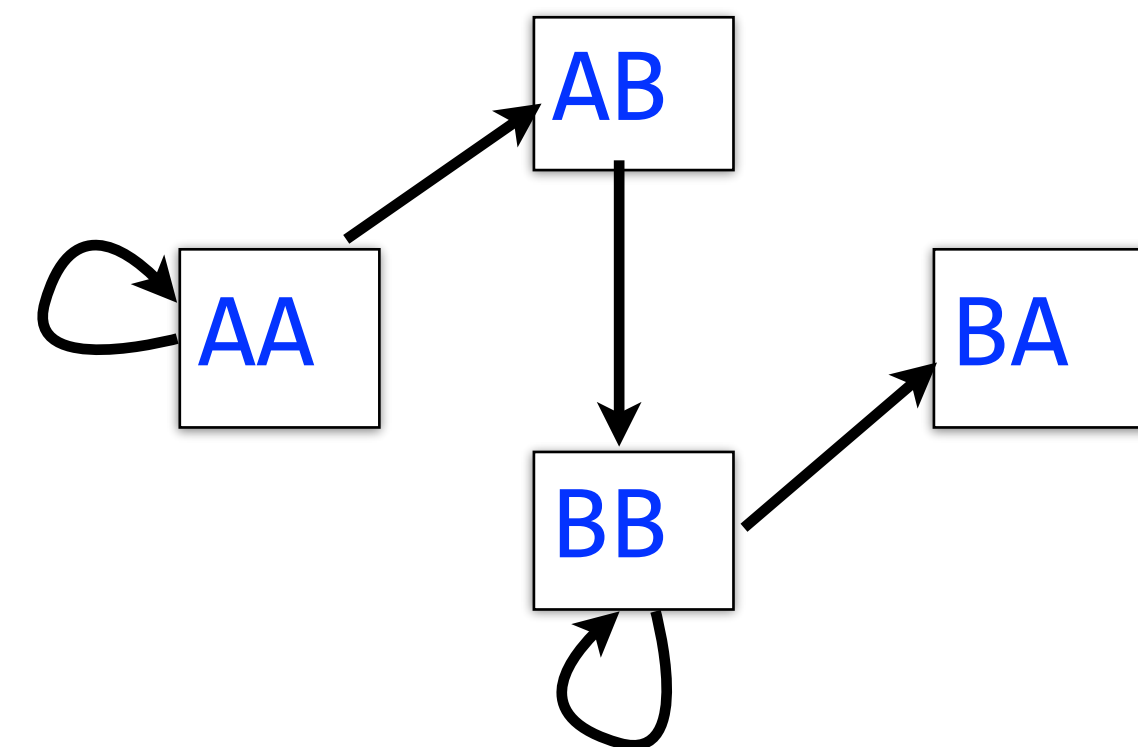
Node is balanced if indegree equals outdegree



# Eulerian walk definitions and statements

Node is balanced if indegree equals outdegree

Node is semi-balanced if indegree differs from outdegree by 1

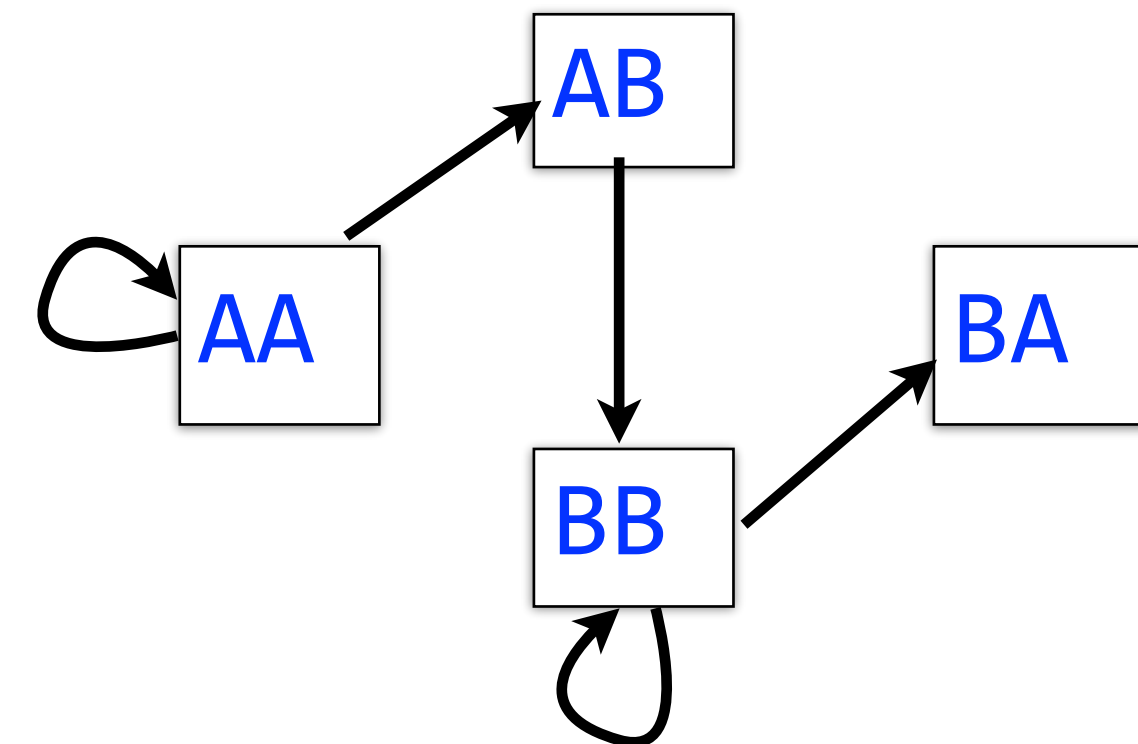


# Eulerian walk definitions and statements

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Graph is connected if each node can be reached by some other node



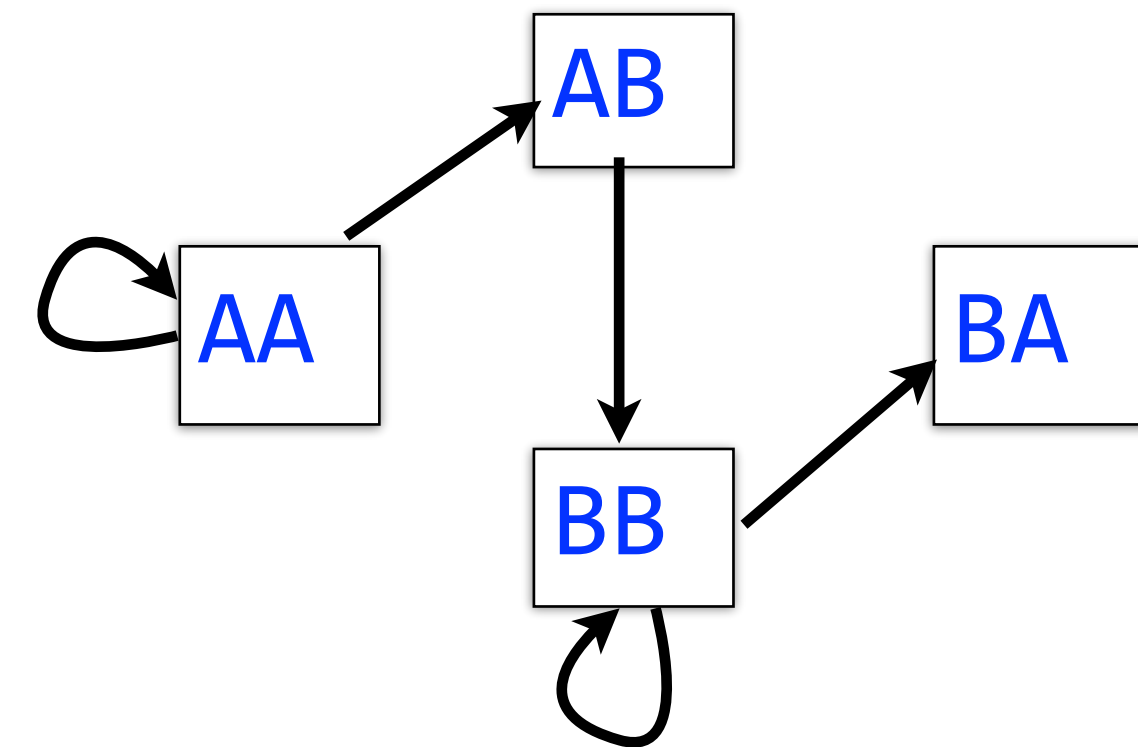
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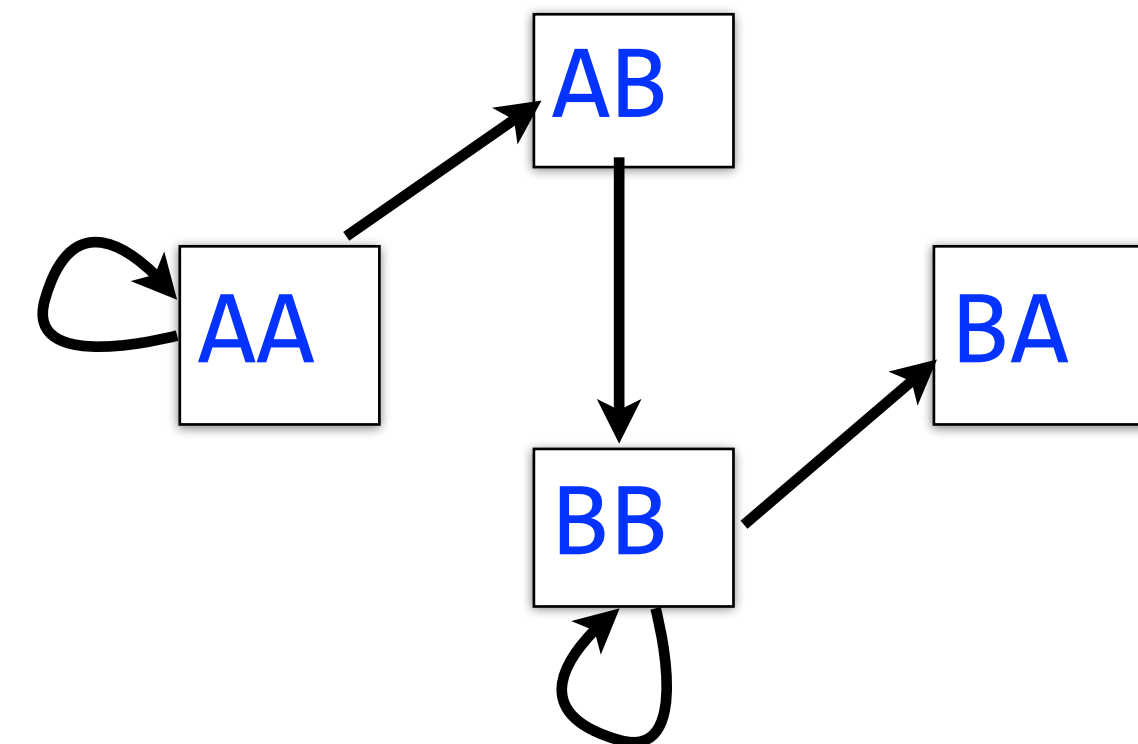
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Not all graphs have Eulerian walks. Graphs that do are Eulerian. (For simplicity, we won't distinguish Eulerian from semi-Eulerian.)



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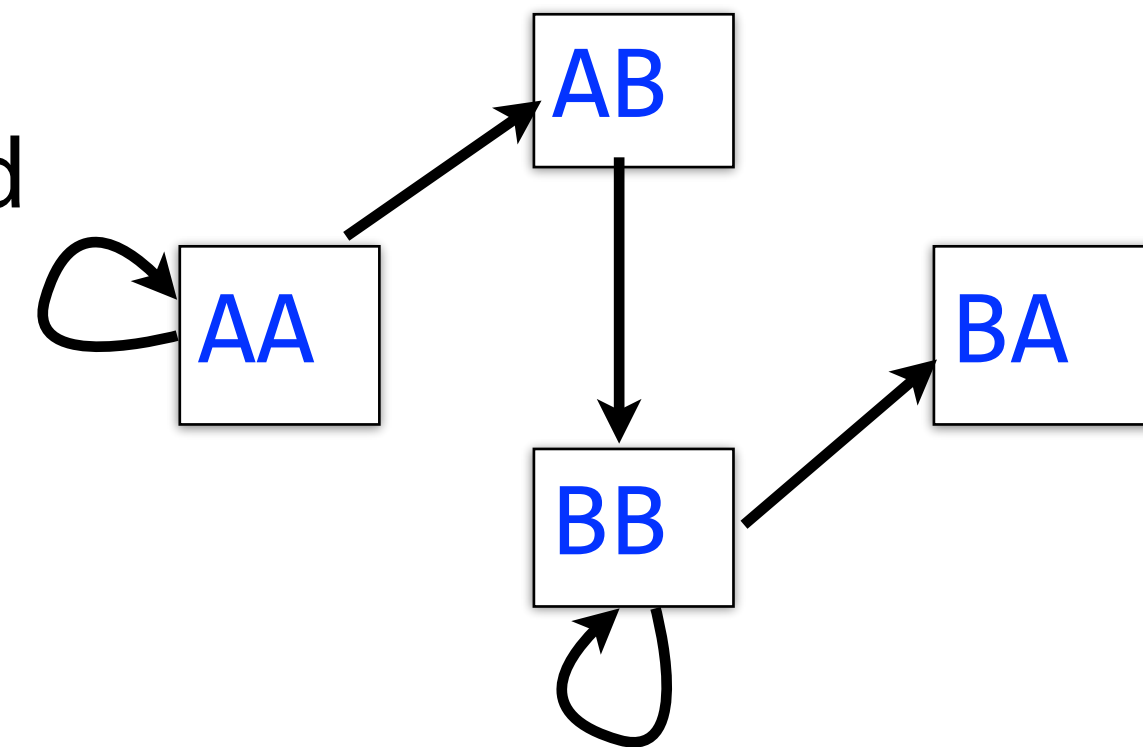
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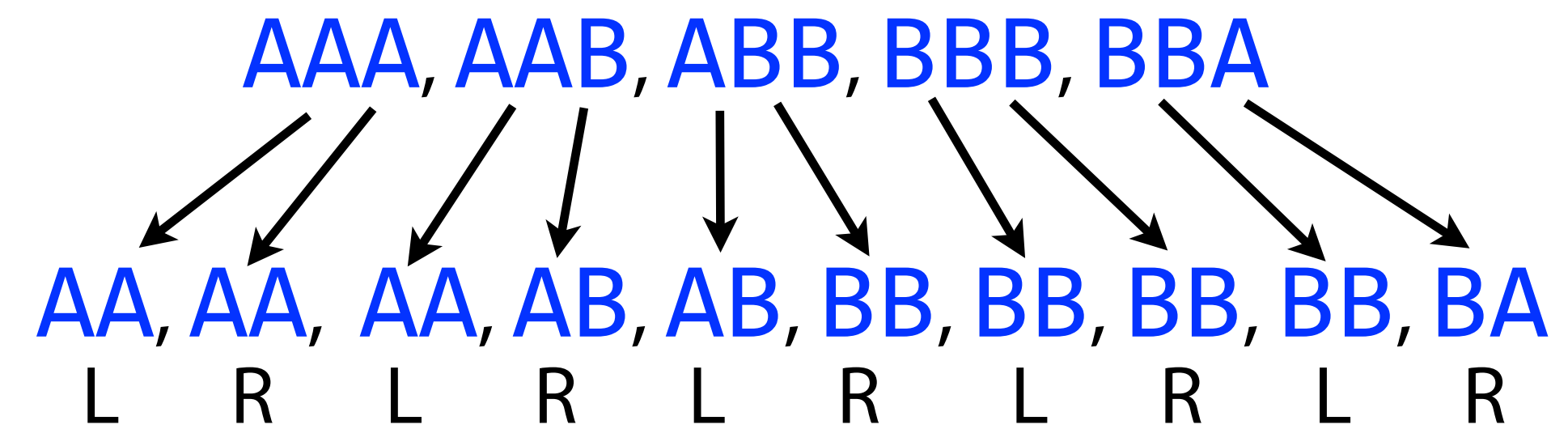
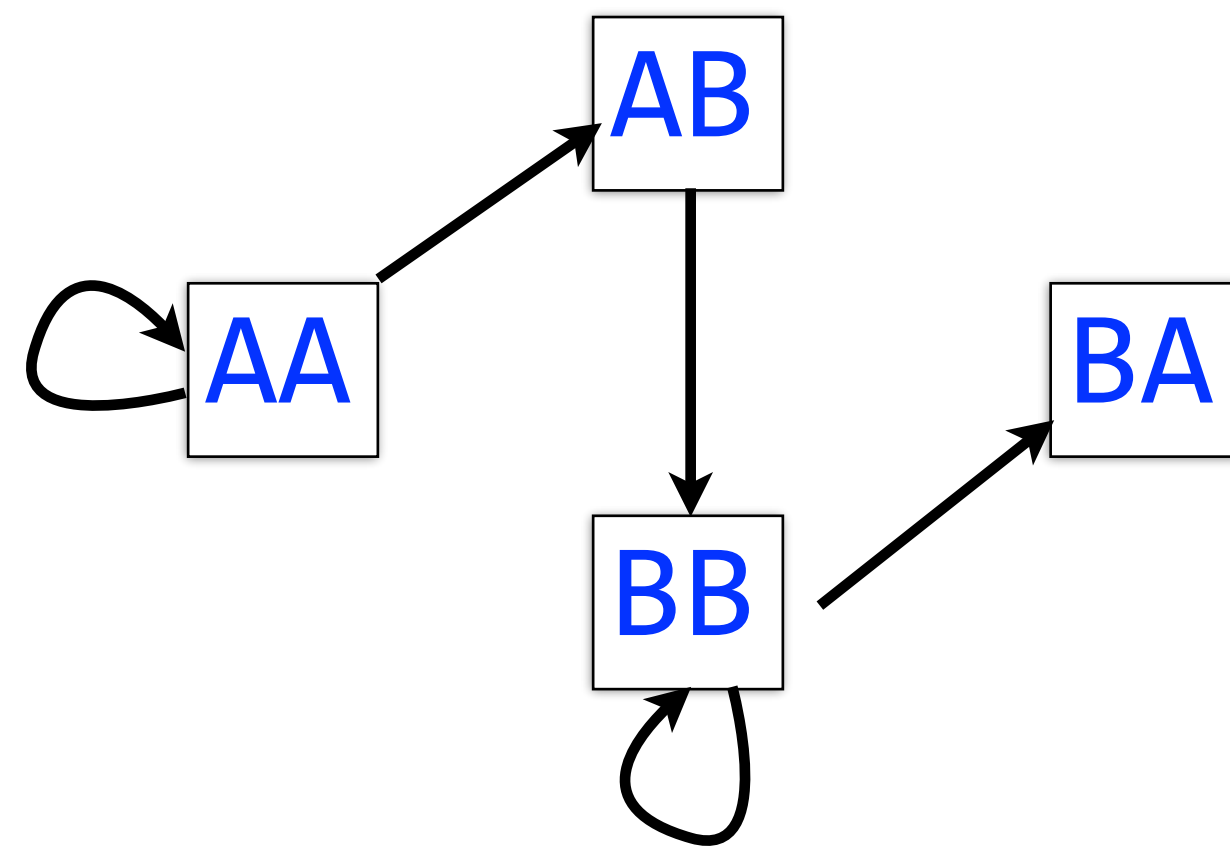
A directed, connected graph is Eulerian if and only if it has at most 2 semi-balanced nodes and all other nodes are balanced

Jones and Pevzner section 8.8



# De Bruijn graph

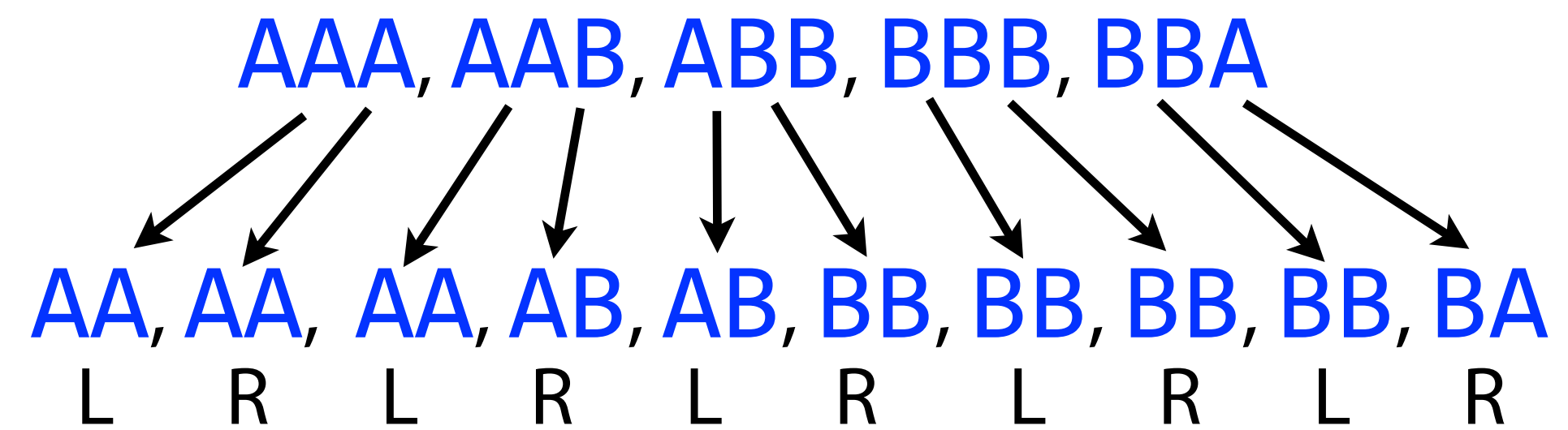
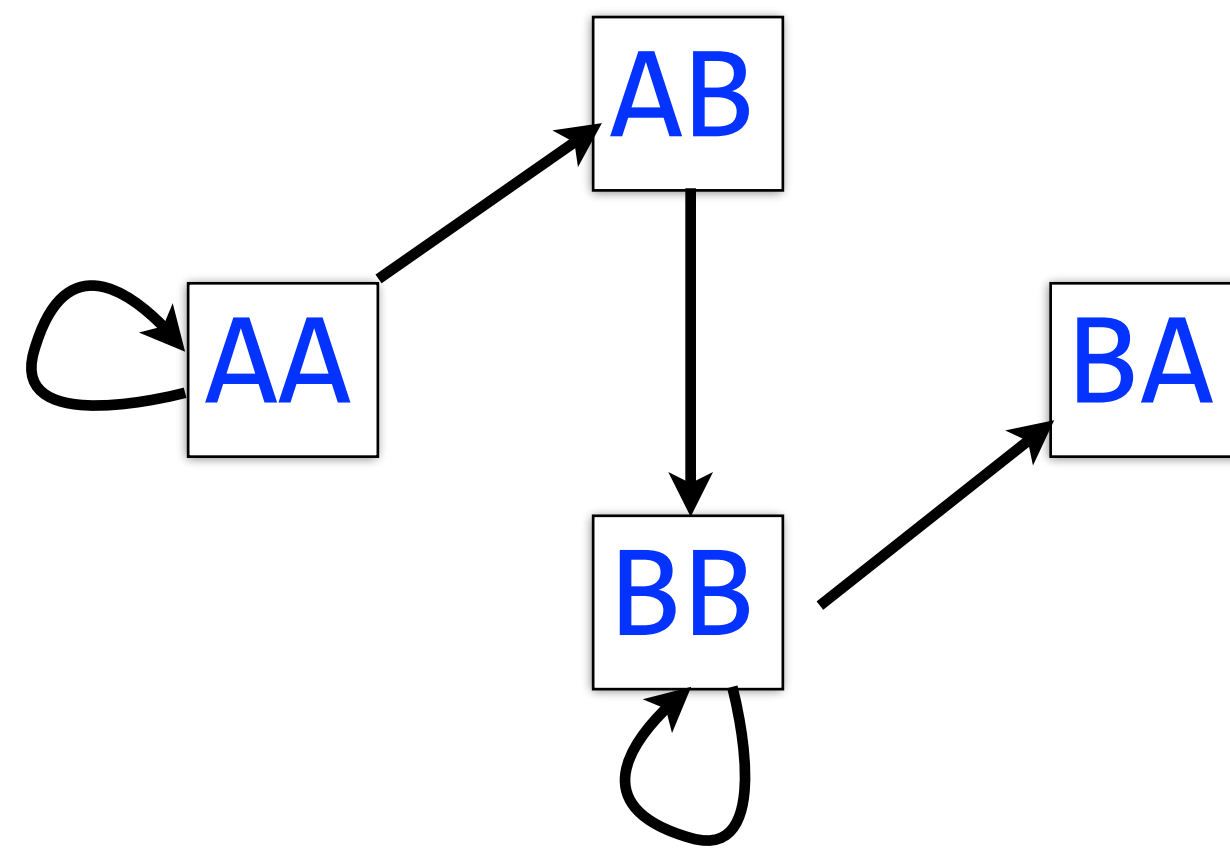
Back to de Bruijn graph



Is it Eulerian?

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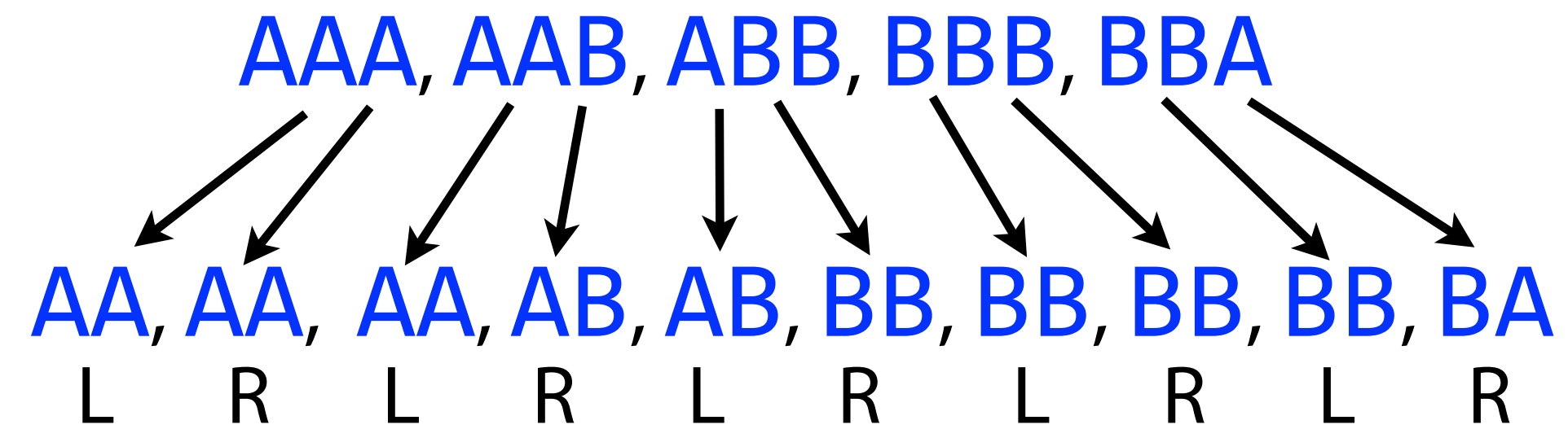
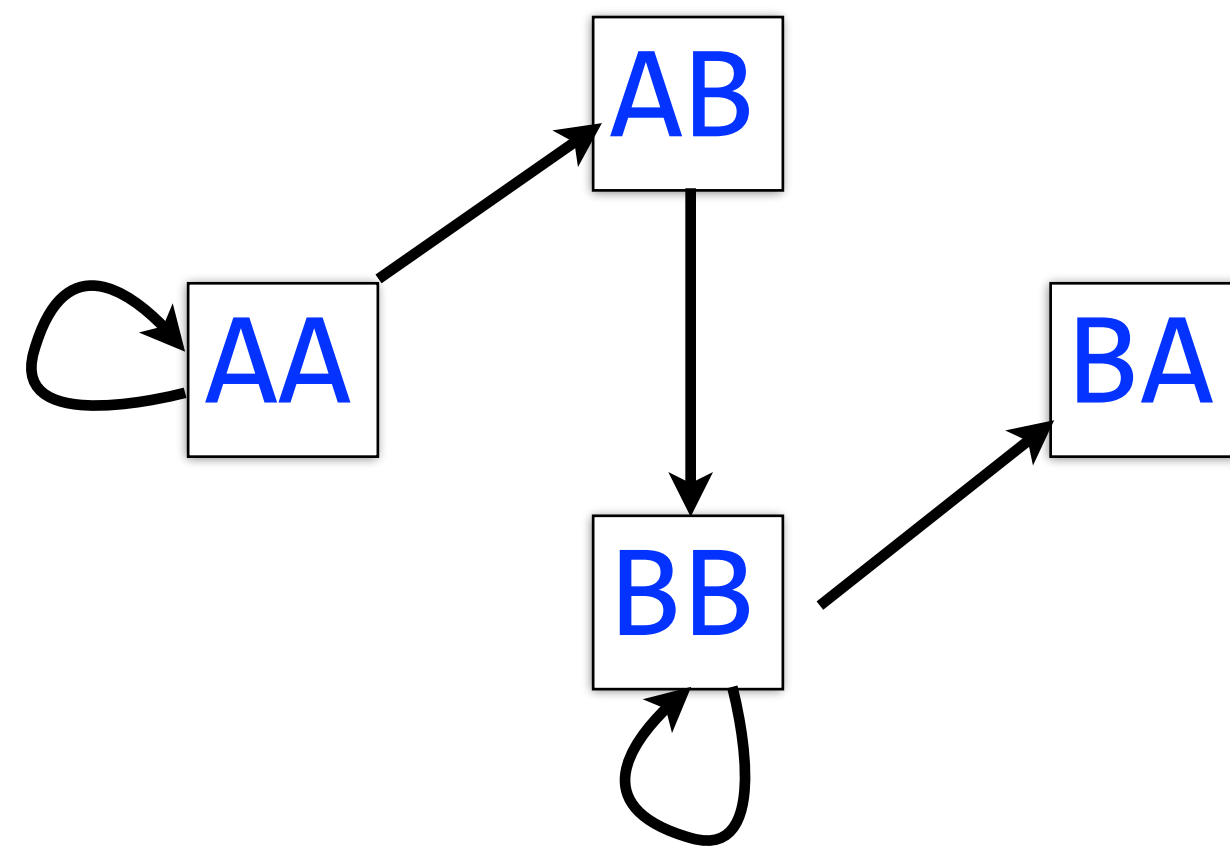
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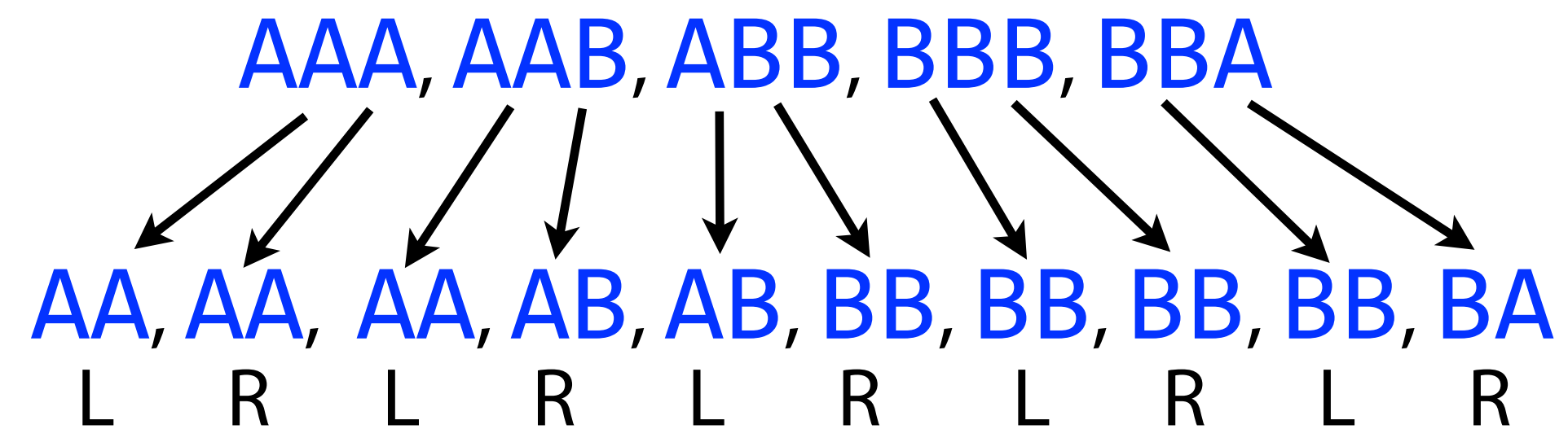
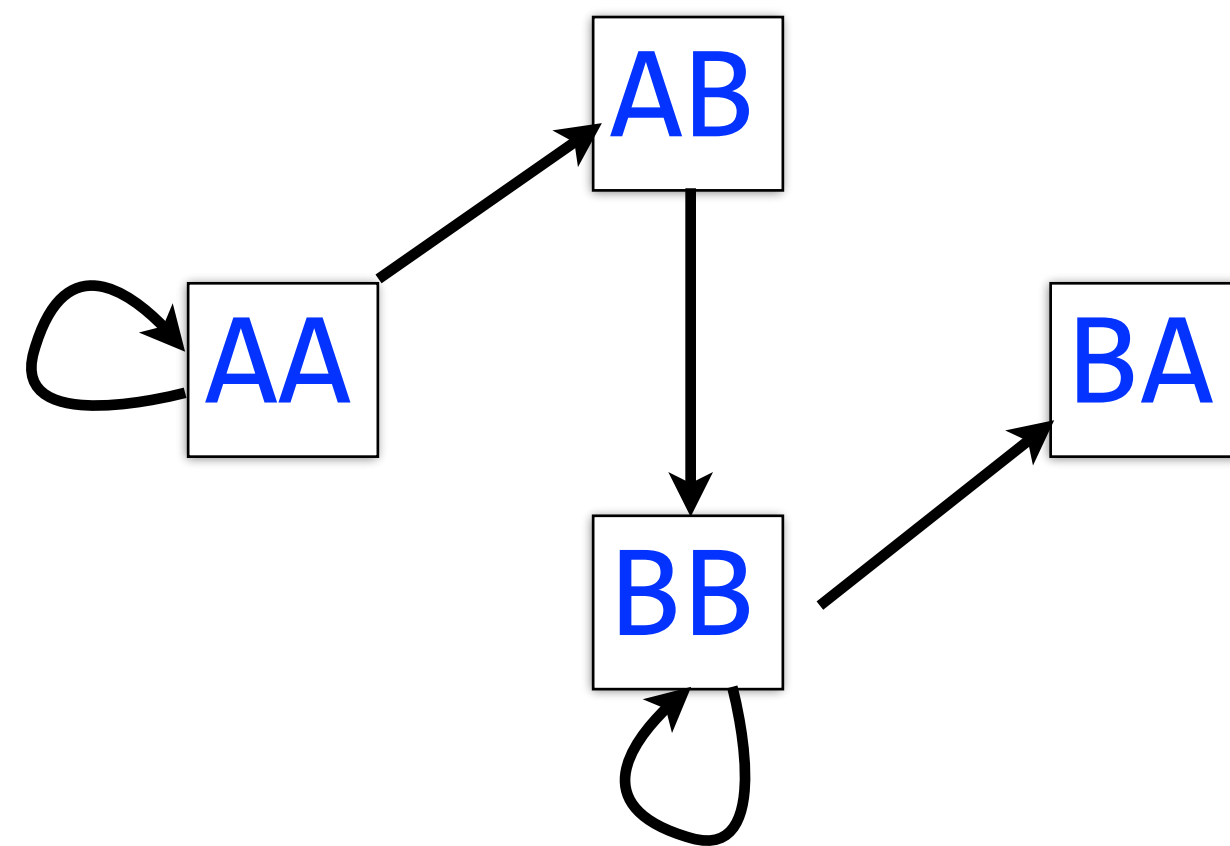


Is it Eulerian? Yes

Argument 1: AA → AA → AB → BB → BB → BA

# De Bruijn graph

Back to de Bruijn graph



Is it Eulerian? Yes

Argument 1:  $AA \rightarrow AA \rightarrow AB \rightarrow BB \rightarrow BB \rightarrow BA$

Argument 2:  $AA$  and  $BA$  are semi-balanced,  $AB$  and  $BB$  are balanced

# De Bruijn graph

A procedure for making a de Bruijn graph for a genome

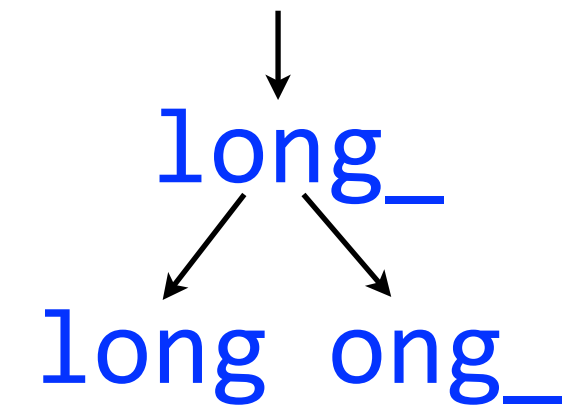
Assume “perfect sequencing”: each genome k-mer is sequenced exactly once with no errors

Pick a substring length k: 5

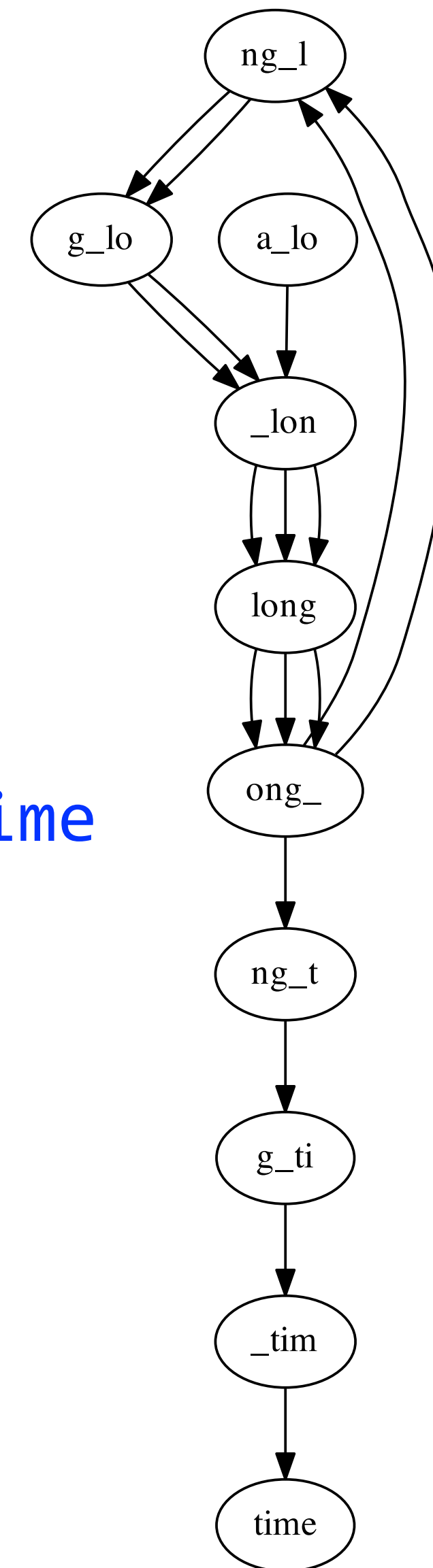
Start with an input string:

a\_long\_long\_long\_time

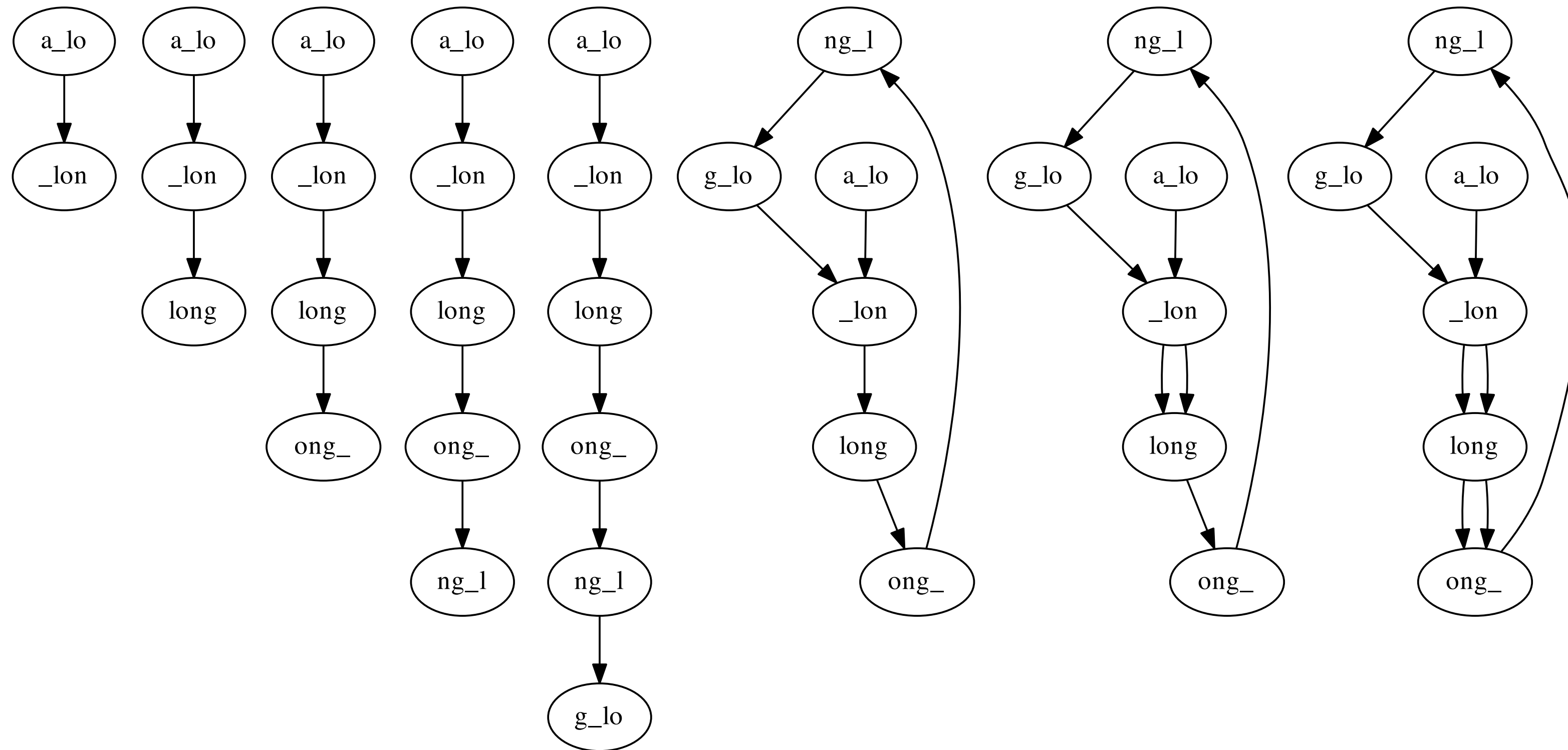
Take each k mer and split into left and right k-1 mers



Add k-1 mers as nodes to de Bruijn graph (if not already there), add edge from left k-1 mer to right k-1 mer



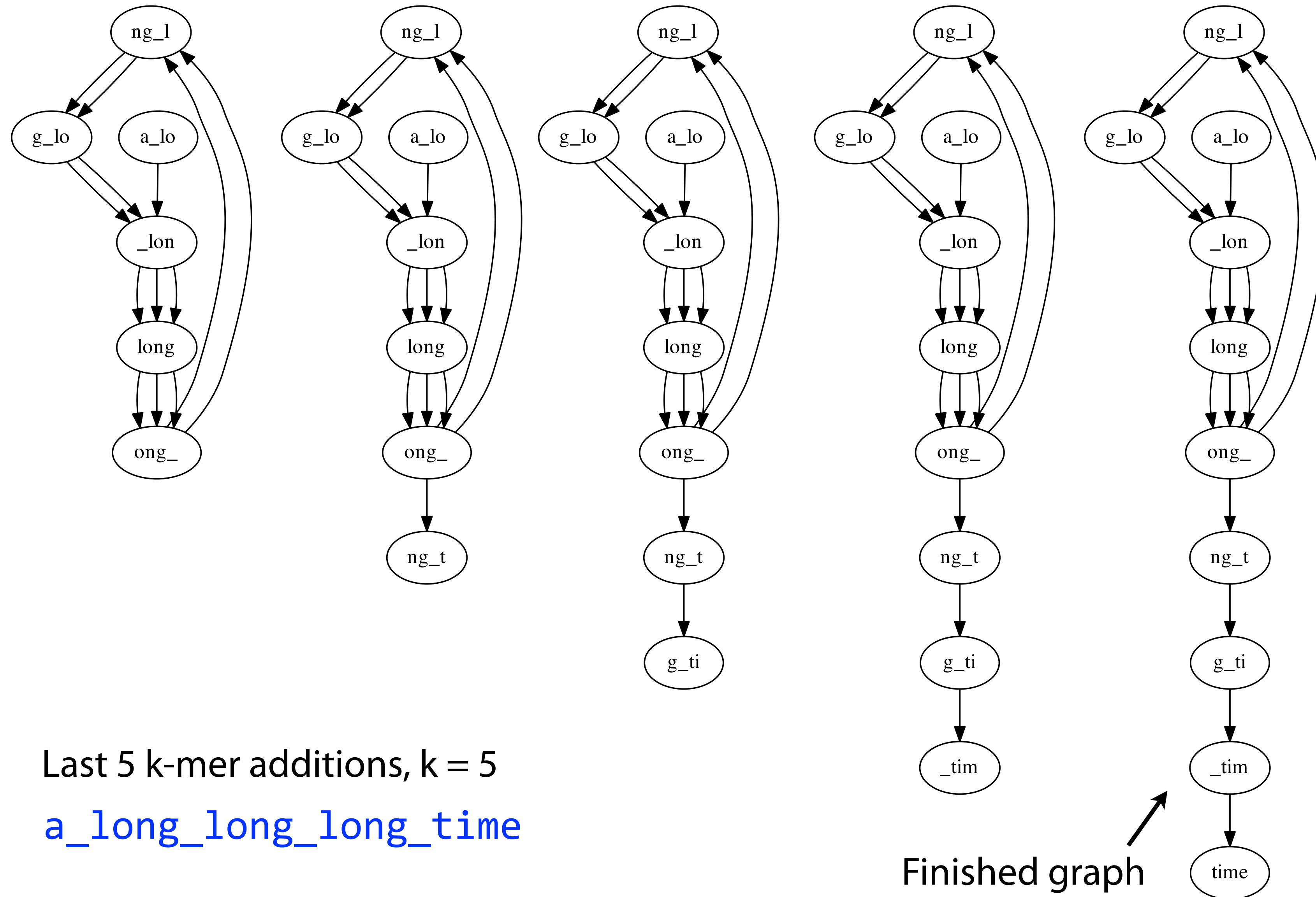
# De Bruijn graph



First 8 k-mer additions,  $k = 5$

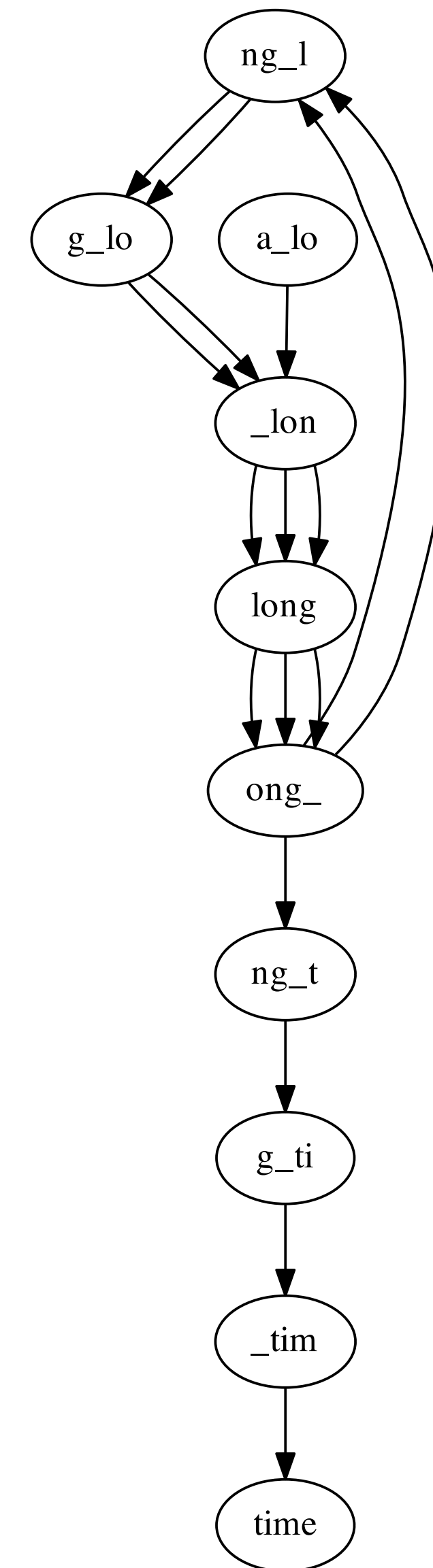
`a_long_long_long_time`

# De Bruijn graph



# De Bruijn graph

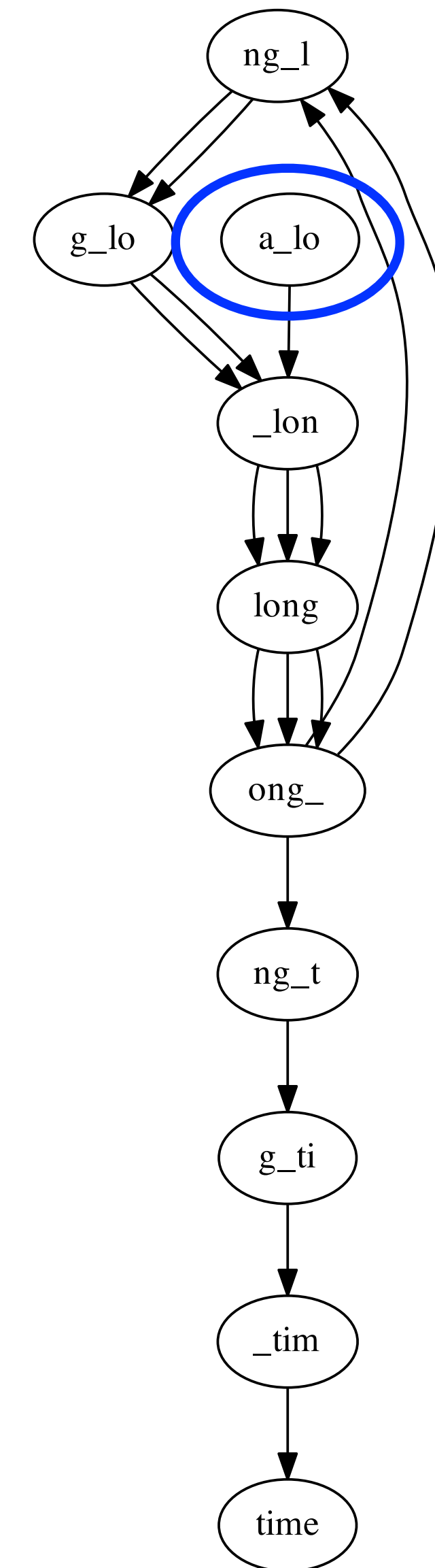
Procedure yields Eulerian graph. Why?



# De Bruijn graph

Procedure yields Eulerian graph. Why?

Node for  $k-1$ -mer from **left end** is semi-balanced with one more outgoing edge than incoming \*



\* Unless left- and right-most  $k-1$ -mers are equal

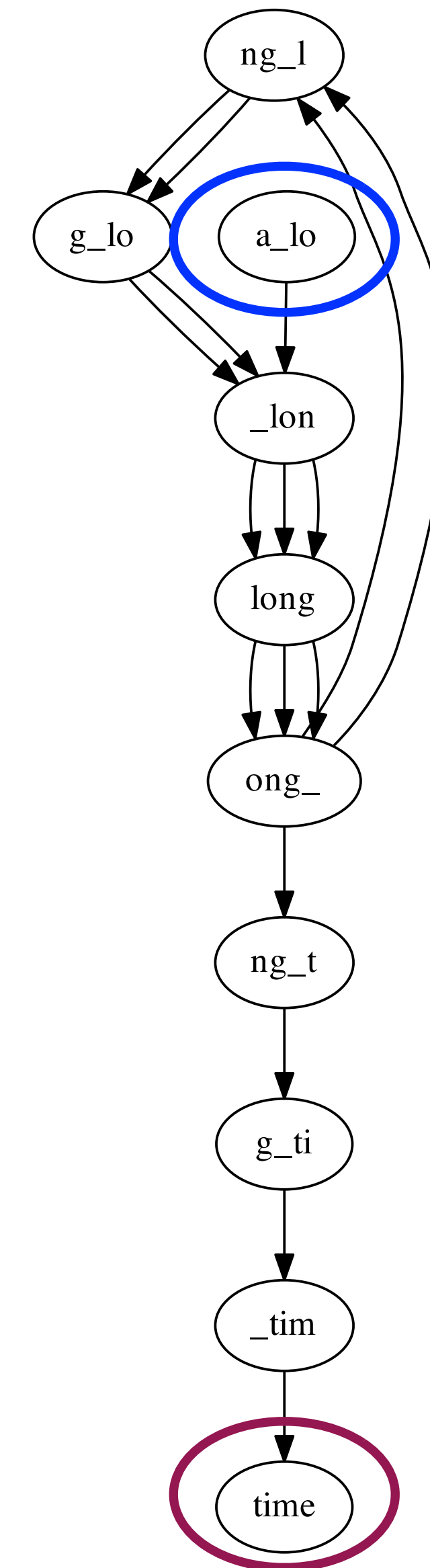
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Node for k-1-mer at **right end** is semi-balanced with one more incoming than outgoing \*

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# De Bruijn graph

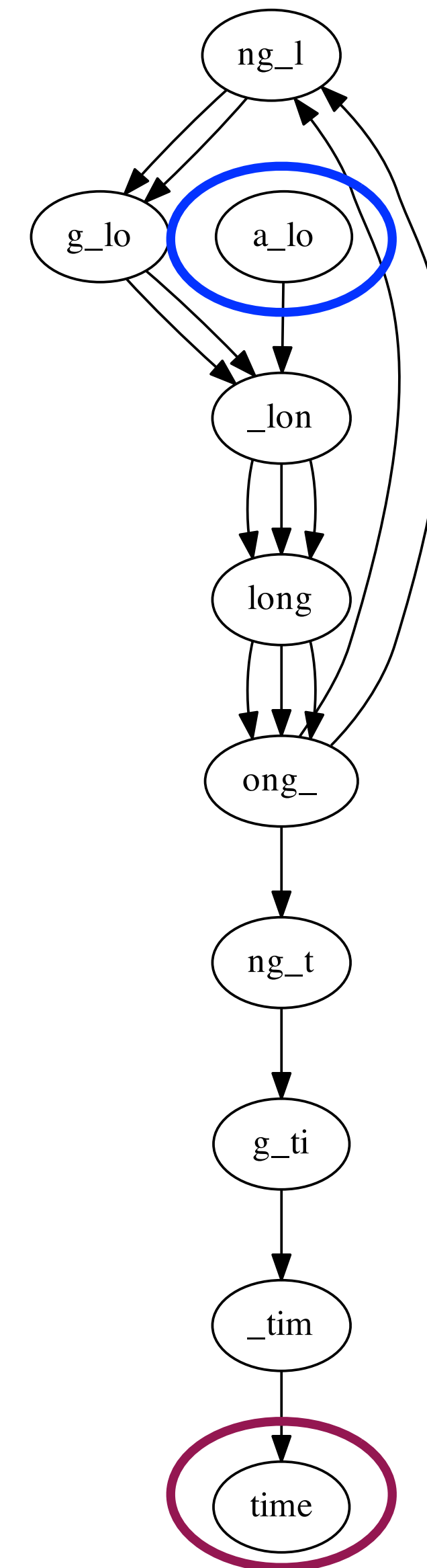
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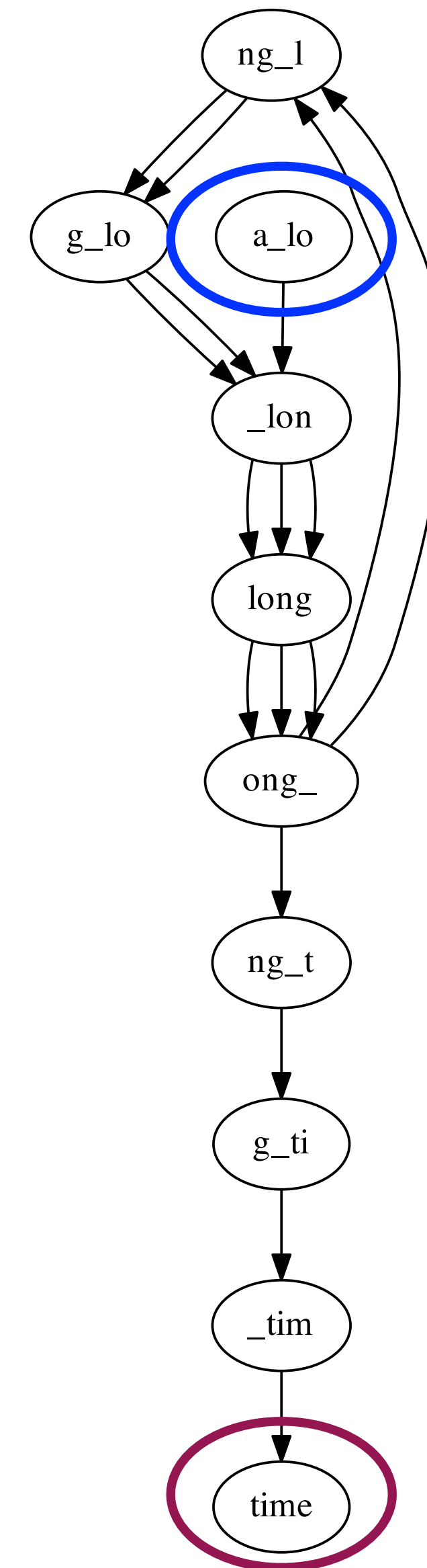
Other nodes are balanced since # times k-1-mer occurs as a left k-1-mer = # times it occurs as a right k-1-mer

\* Unless left- and right-most k-1-mers are equal



# De Bruijn graph

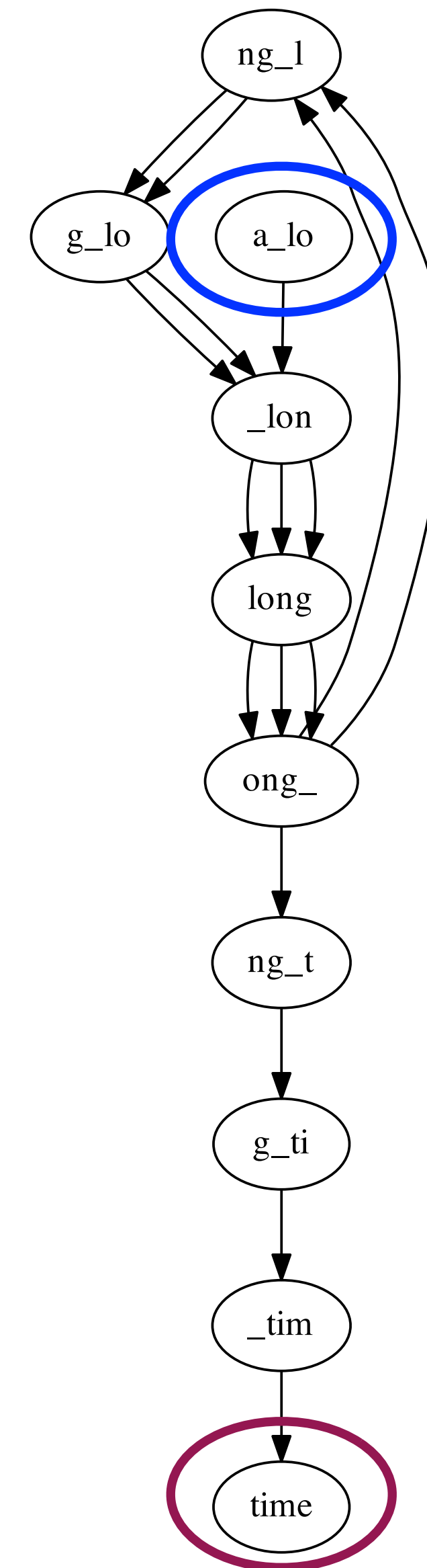
What string does the Eulerian path spell out?



# De Bruijn graph

What string does the Eulerian path spell out?

a\_long\_long\_long\_time

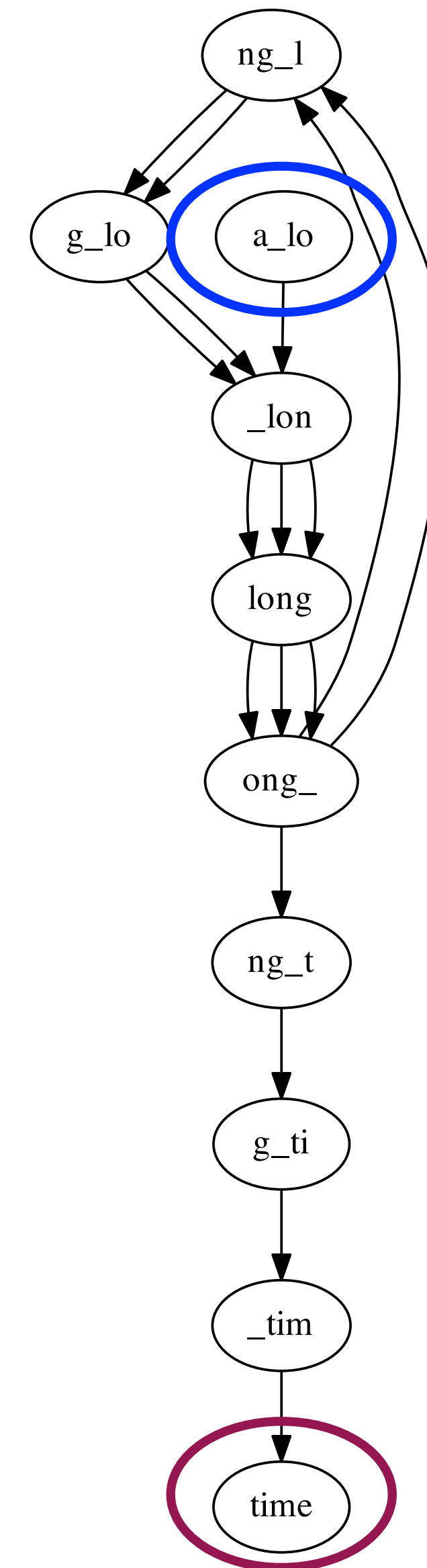


# De Bruijn graph

What string does the Eulerian path spell out?

a\_long\_long\_long\_time

The original string! No collapsing!



# De Bruijn graph builder implementation

```
class DeBruijnGraph:
    """ A de Bruijn multigraph built from a collection of strings.
        User supplies strings and k-mer length k. Nodes of the de
        Bruijn graph are k-1-mers and edges join a left k-1-mer to a
        right k-1-mer. """

    @staticmethod
    def chop(st, k):
        """ Chop a string up into k mers of given length """
        for i in xrange(0, len(st)-(k-1)): yield st[i:i+k]

    class Node:
        """ Node in a de Bruijn graph, representing a k-1 mer """
        def __init__(self, km1mer):
            self.km1mer = km1mer

        def __hash__(self):
            return hash(self.km1mer)

    def __init__(self, strIter, k):
        """ Build de Bruijn multigraph given strings and k-mer length k """
        self.G = {} # multimap from nodes to neighbors
        self.nodes = {} # maps k-1-mers to Node objects
        self.k = k
        for st in strIter:
            for kmer in self.chop(st, k):
                km1L, km1R = kmer[:-1], kmer[1:]
                nodeL, nodeR = None, None
                if km1L in self.nodes:
                    nodeL = self.nodes[km1L]
                else:
                    nodeL = self.nodes[km1L] = self.Node(km1L)
                if km1R in self.nodes:
                    nodeR = self.nodes[km1R]
                else:
                    nodeR = self.nodes[km1R] = self.Node(km1R)
                self.G.setdefault(nodeL, []).append(nodeR)
```

Chop string into k-mers

For each k-mer, find left  
and right k-1-mers

Create corresponding  
nodes (if necessary) and  
add edge

# De Bruijn graph

For Eulerian graph, Eulerian walk can be found in  $O(|E|)$  time.  $|E|$  is # edges.

Convert graph into one with Eulerian cycle (add an edge to make all nodes balanced), then use this recursive procedure

Insight: If  $C$  is a cycle in an Eulerian graph, then after removing edges of  $C$ , remaining connected components are also Eulerian

```
# Make all nodes balanced, if not already

tour = []
# Pick arbitrary node
src = g.iterkeys().next()

def __visit(n):
    while len(g[n]) > 0:
        dst = g[n].pop()
        __visit(dst)
        tour.append(n)

__visit(src)
# Reverse order, omit repeated node
tour = tour[::-1][:-1]

# Turn tour into walk, if necessary
```

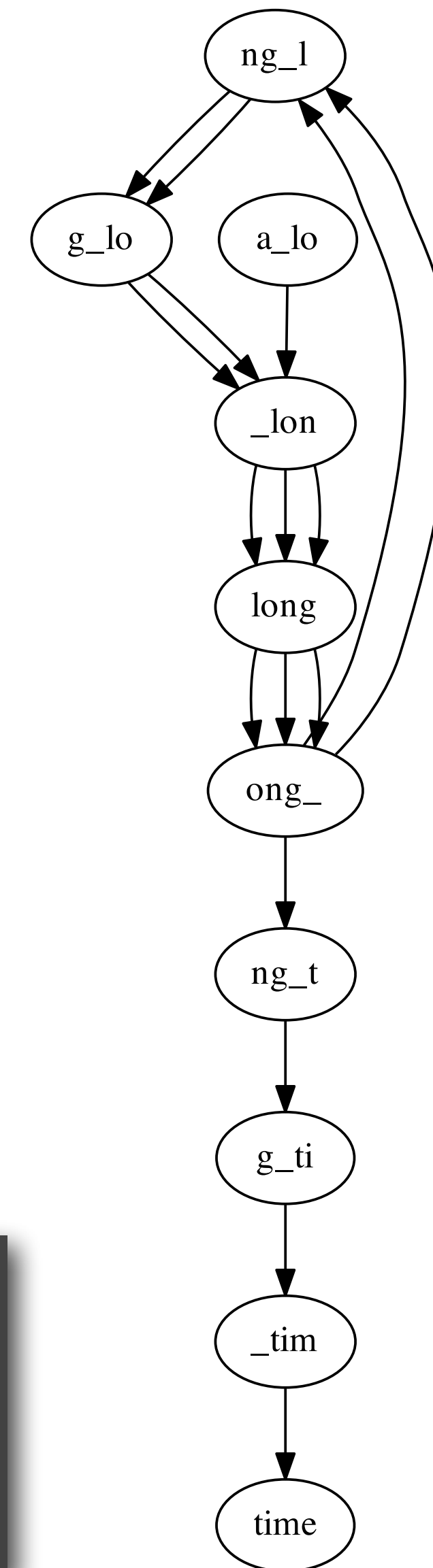
# De Bruijn graph

Full illustrative de Bruijn graph and Eulerian walk implementation:

[http://bit.ly/CG\\_DeBruijn](http://bit.ly/CG_DeBruijn)

Example where Eulerian walk gives correct answer for small k whereas Greedy-SCS could spuriously collapse repeat:

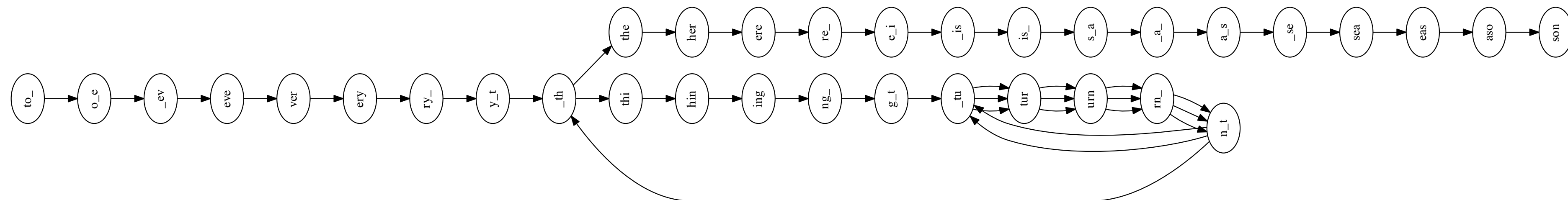
```
>>> G = DeBruijnGraph(["a_long_long_long_time"], 5)
>>> print G.eulerianWalkOrCycle()
['a_lo', '_lon', 'long', 'ong_', 'ng_l', 'g_lo',
 '_lon', 'long', 'ong_', 'ng_l', 'g_lo', '_lon', 'long',
 'ong_', 'ng_t', 'g_ti', '_tim', 'time']
```



# De Bruijn graph

```
>>> st = "to_everything_turn_turn_turn_there_is_a_season"  
>>> G = DeBruijnGraph([st], 4)  
>>> path = G.eulerianWalkOrCycle() # Fast! Linear in # edges  
>>> superstring = path[0] + ''.join(map(lambda x: x[-1], path[1:]))  
>>> print superstring  
to_everything_turn_turn_turn_there_is_a_season
```

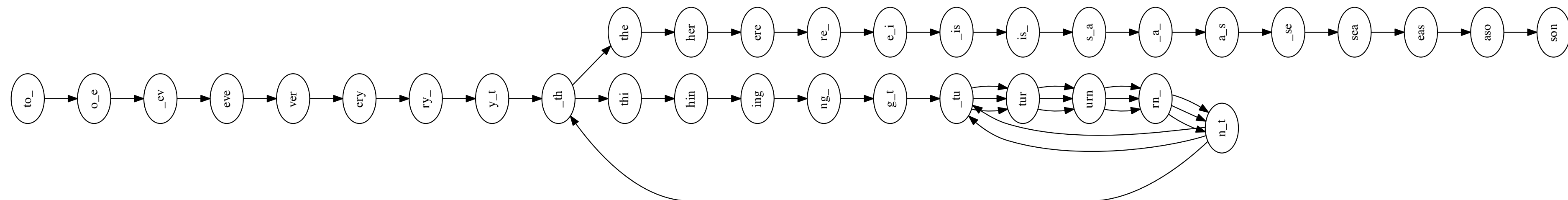
[http://bit.ly/CG\\_DeBruijn](http://bit.ly/CG_DeBruijn)



# De Bruijn graph

```
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to_everything_turn_turn_turn_there_is_a_season
```

[http://bit.ly/CG\\_DeBruijn](http://bit.ly/CG_DeBruijn)

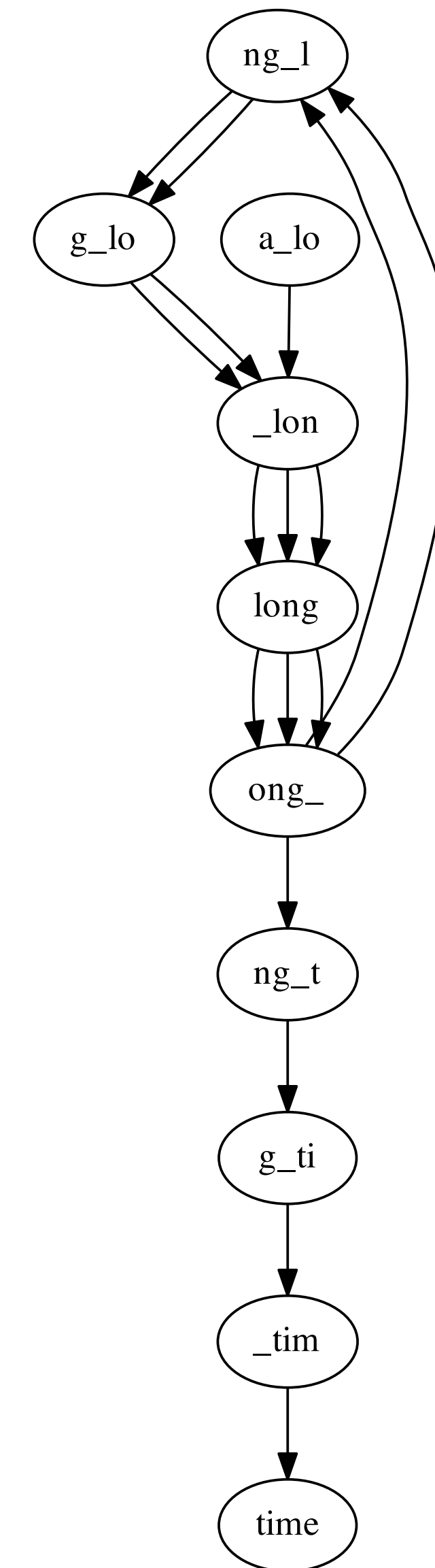


Recall: This is not generally possible or tractable in the overlap/SCS formulation

# De Bruijn graph

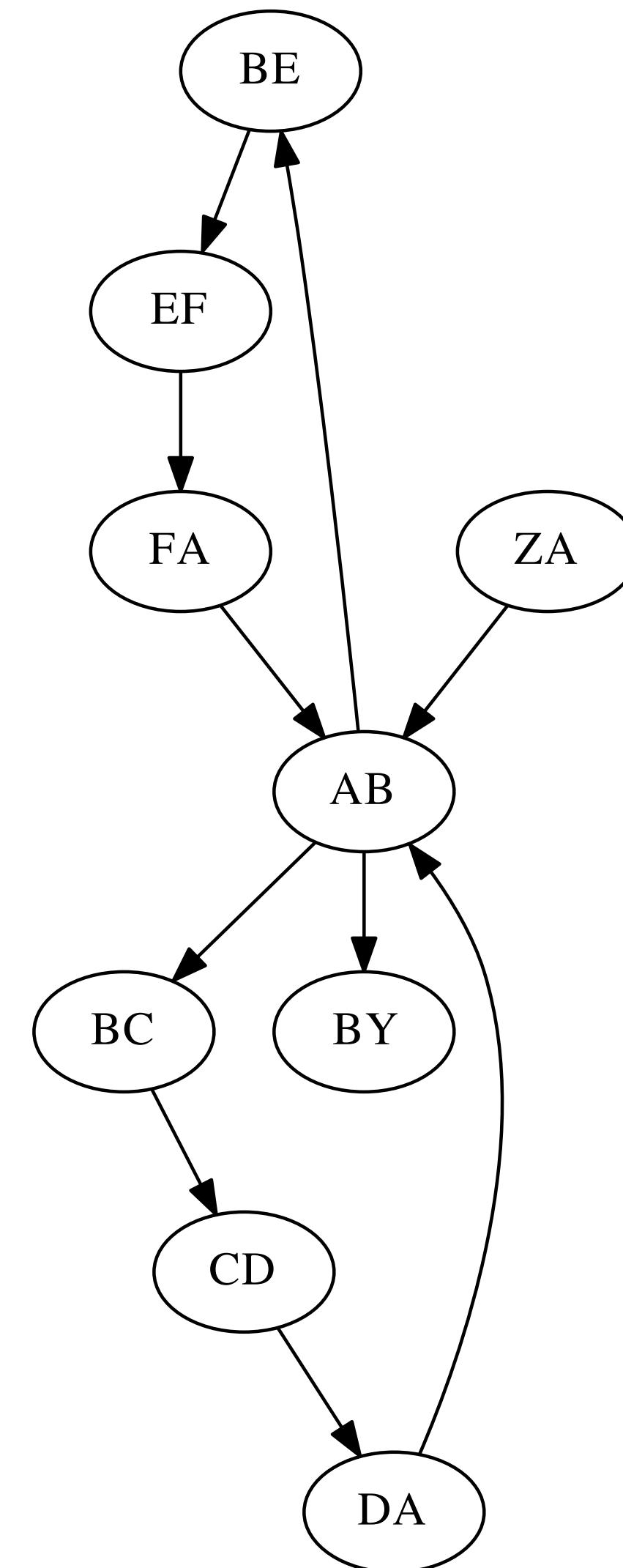
Assuming perfect sequencing, procedure yields graph with Eulerian walk that can be found efficiently.

We saw cases where Eulerian walk corresponds to the original superstring. Is this always the case?



# De Bruijn graph

Problem 1: Repeats still cause misassemblies

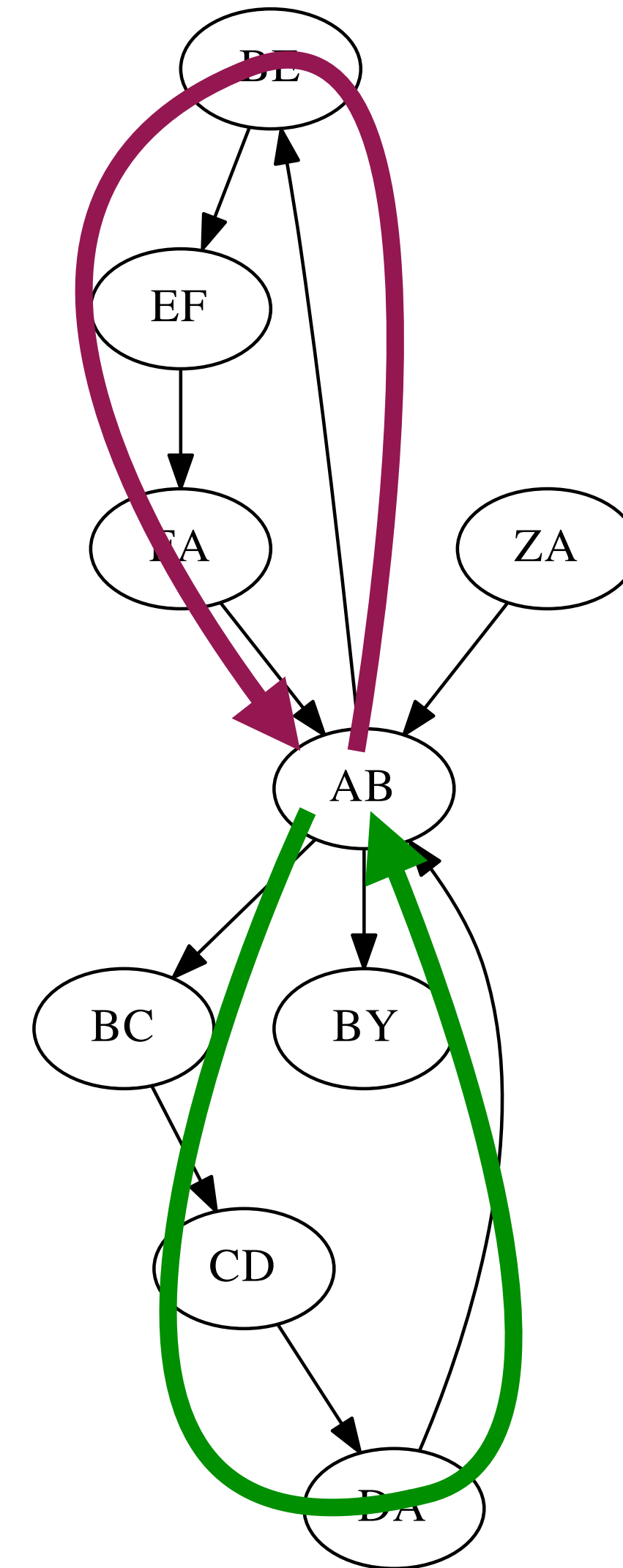


# De Bruijn graph

Problem 1: Repeats still cause misassemblies

ZA → AB → BE → EF → FA → AB → BC → CD → DA → AB → BY

ZA → AB → BC → CD → DA → AB → BE → EF → FA → AB → BY



# De Bruijn graph

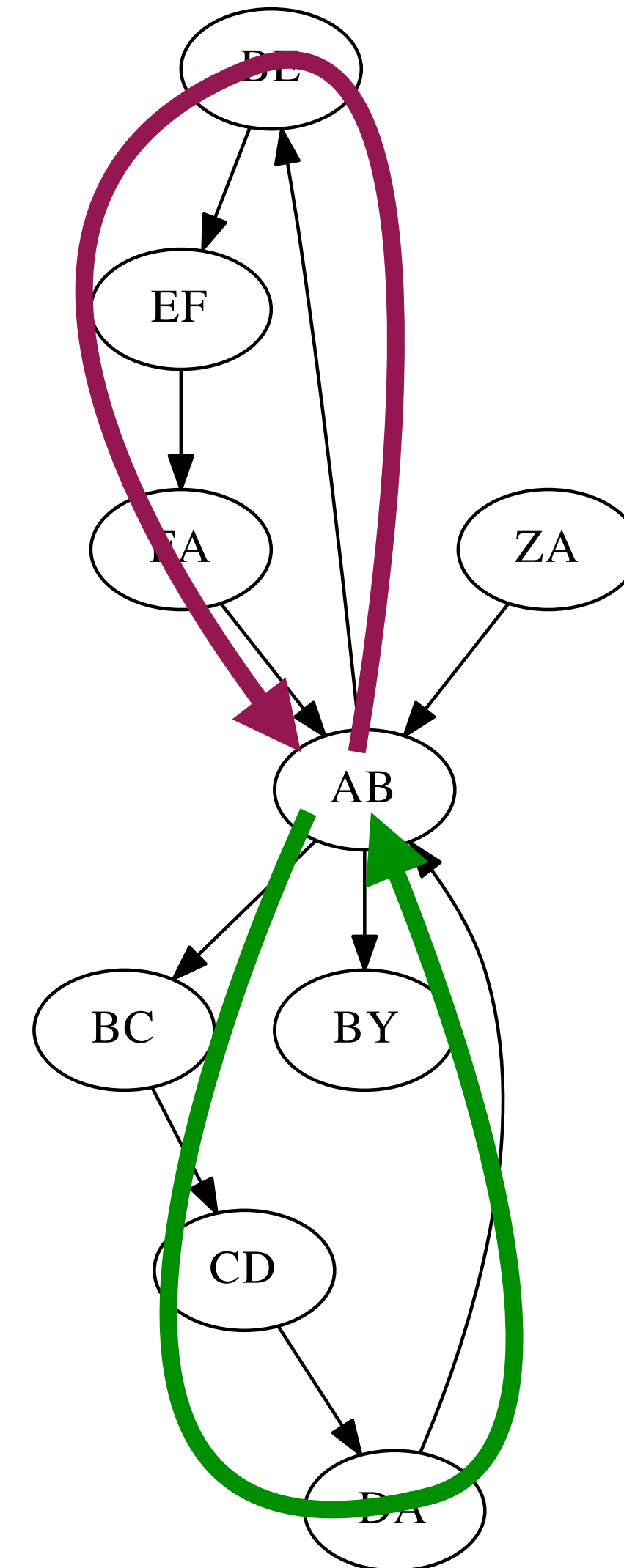
Problem 1: Repeats still cause misassemblies

ZA → AB → BE → EF → FA → AB → BC → CD → DA → AB → BY

ZA → AB → BC → CD → DA → AB → BE → EF → FA → AB → BY

Problem 2:

We've been building DBGs assuming "perfect" sequencing: each k-mer reported exactly once, no mistakes. Real datasets aren't like that.



# The Problem with Eulerian Paths

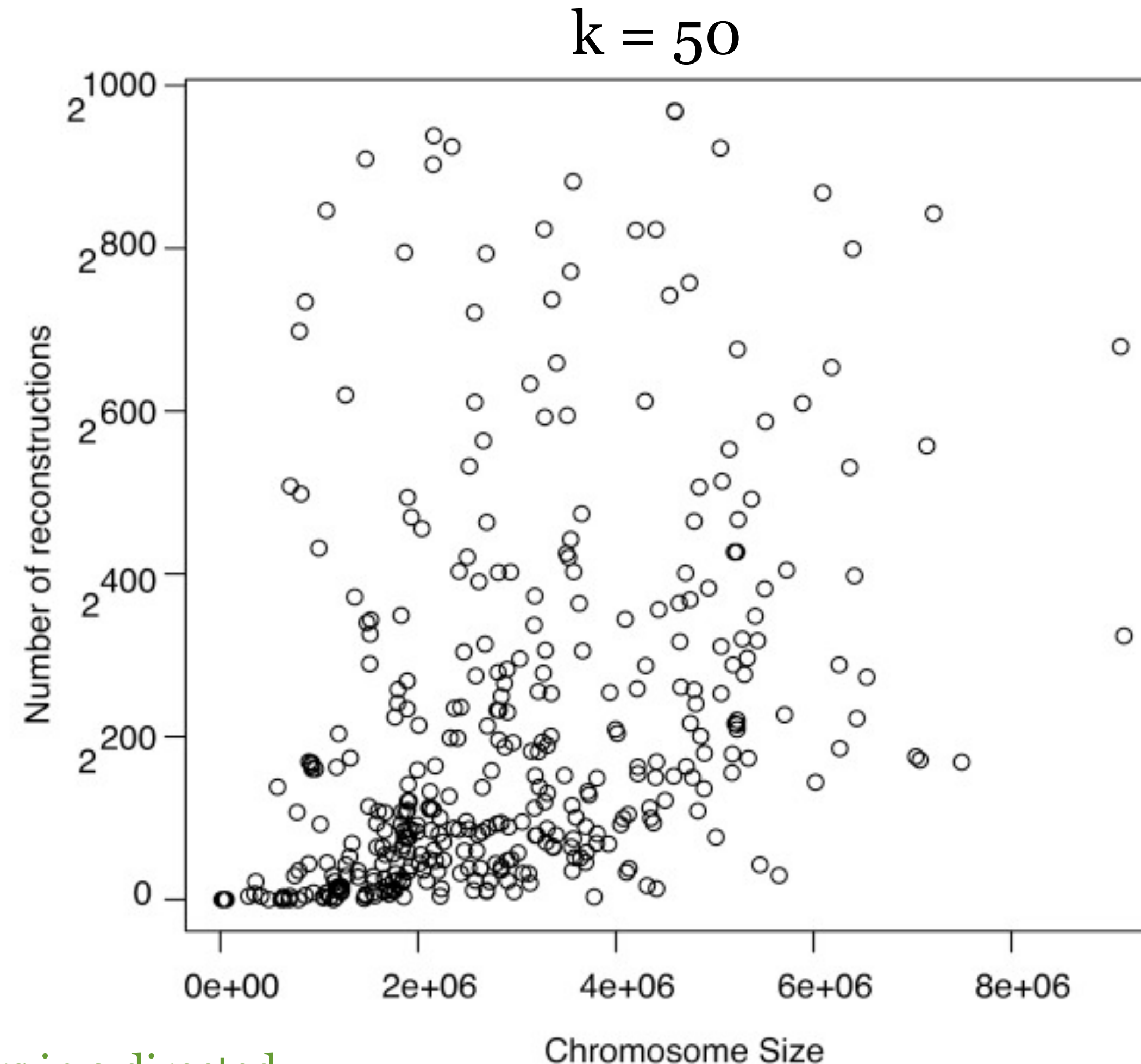
There are typically an astronomical number of possible Eulerian tours with perfect data.

Adding back constraints to limit # of tours leads to a NP-hard problem.

With imperfect data, there are usually NO Eulerian tours

Estimating # of parallel edges is usually tricky.

Aside: counting # of Eulerian tours in a directed graph is easy, but in an undirected graph is #P-complete (hard).



(Kingsford, Schatz, Pop, 2010)

\* slide courtesy of Carl Kingsford

# Third law of assembly

Repeats make assembly difficult; whether we can assemble without mistakes depends on length of reads and repetitive patterns in genome

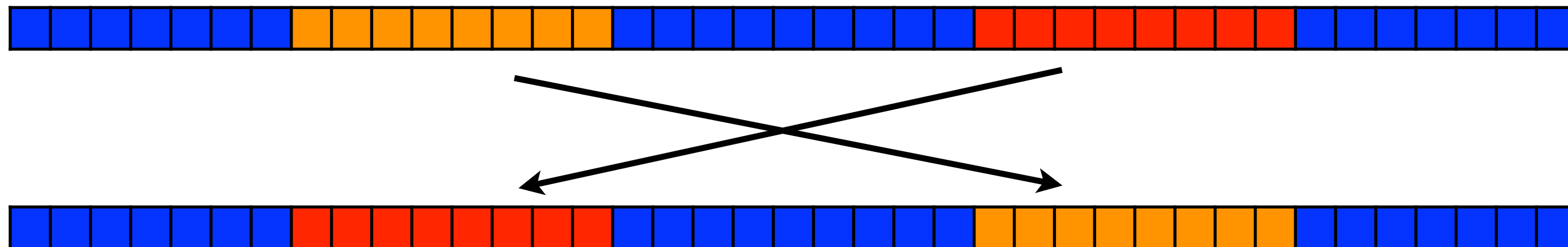
Collapsing:

a\_long\_long\_long\_time



a\_long\_long\_time

Shuffling:

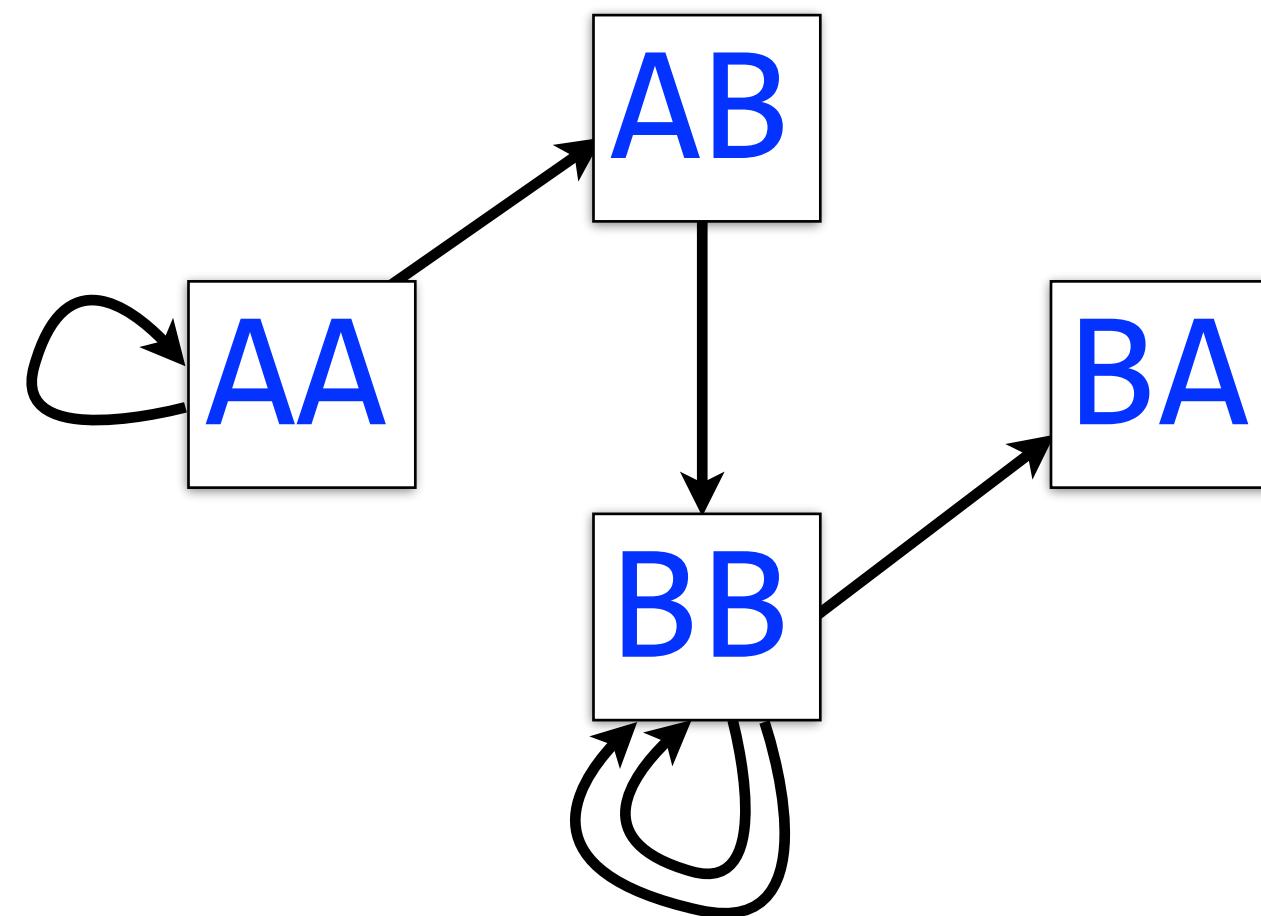


# De Bruijn graph

genome: **AAABBBBA**

3-mers: **AAA, AAB, ABB, BBB, BBB, BBA**

L/R 2-mers: **AA, AA    AA, AB    AB, BB    BB, BB    BB, BB    BB, BA**



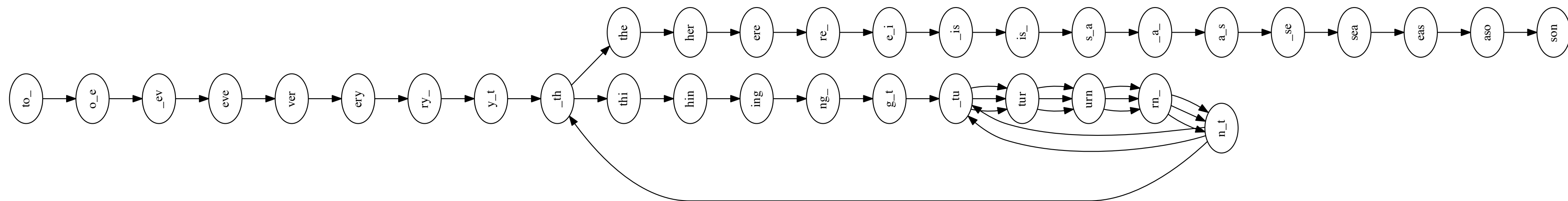
One edge per k-mer

One node per distinct k-1-mer

# De Bruijn graph

```
>>> st = "to_everything_turn_turn_turn_there_is_a_season"
>>> G = DeBruijnGraph([st], 4)
>>> path = G.eulerianWalkOrCycle() # Fast! Linear in # edges
>>> superstring = path[0] + ''.join(map(lambda x: x[-1], path[1:]))
>>> print superstring
to_everything_turn_turn_turn_there_is_a_season
```

[http://bit.ly/CG\\_DeBruijn](http://bit.ly/CG_DeBruijn)



# De Bruijn graph

Case where  $k = 4$  works:

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>>> superstring = path[0] + ''.join(map(lambda x: x[-1], path[1:]))
>>> print superstring
to_every_thing_turn_turn_turn_there_is_a_season
```

# De Bruijn graph

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>>> print superstring
to_every_thing_turn_turn_turn_there_is_a_season
```

But  $k = 3$  does not:

```
>>> st = "to_every_thing_turn_turn_turn_there_is_a_season"
>>> G = DeBruijnGraph([st], 3)
>>> path = G.eulerianWalkOrCycle()
>>> superstring = path[0] + ''.join(map(lambda x: x[-1], path[1:]))
>>> print superstring
```

# De Bruijn graph

Case where  $k = 4$  works:

```
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>>> G = DeBruijnGraph([st], 4)
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```

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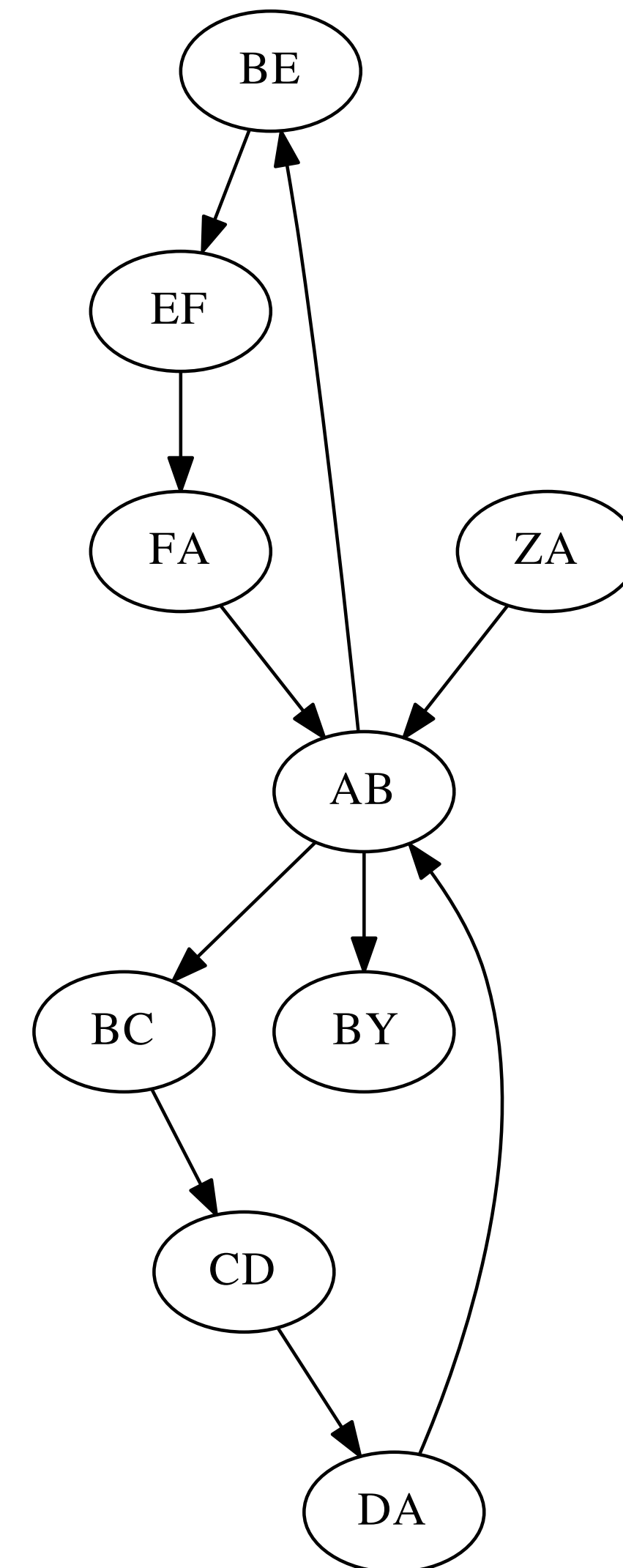
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>>> superstring = path[0] + ''.join(map(lambda x: x[-1], path[1:]))
>>> print superstring
to_every_turn_turn_thing_turn_there_is_a_season
```



Due to repeats that are unresolvable at  $k = 3$

# De Bruijn graph

Problem 1: Repeats still cause misassemblies

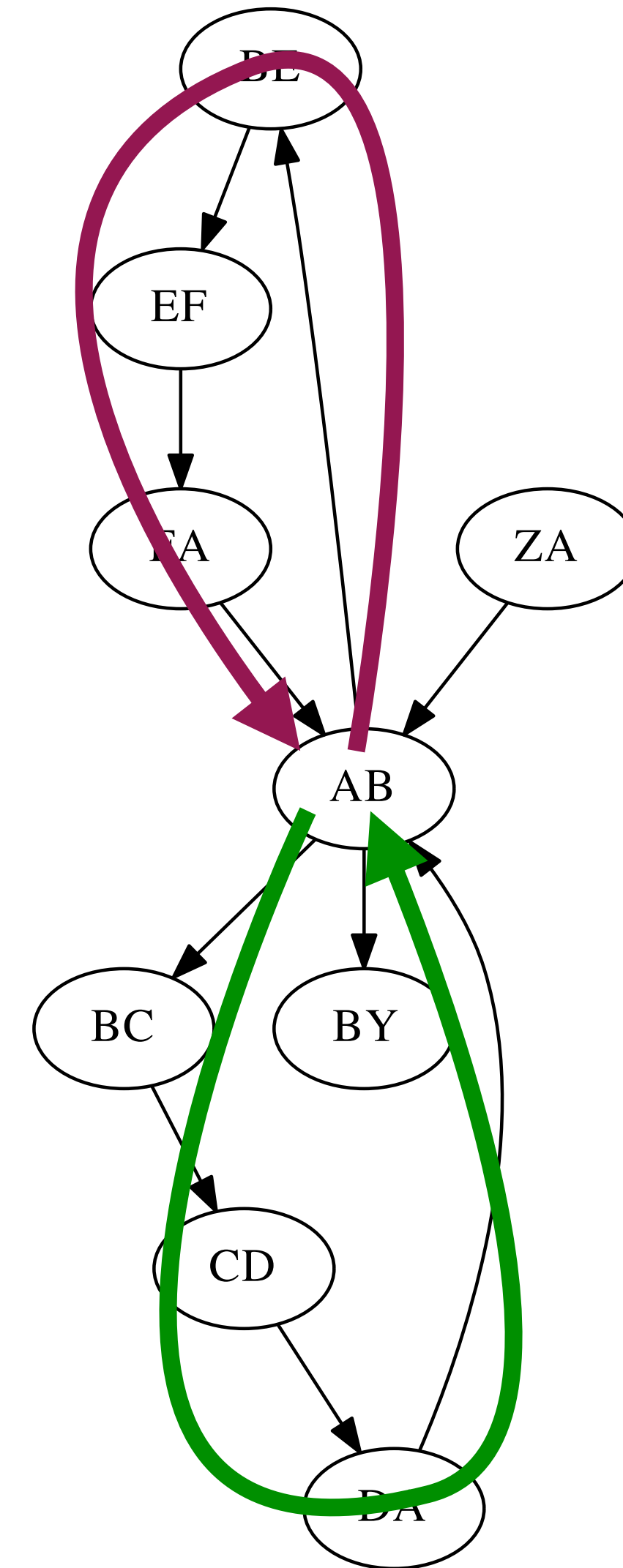


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ZA → AB → BE → EF → FA → AB → BC → CD → DA → AB → BY

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# De Bruijn graph

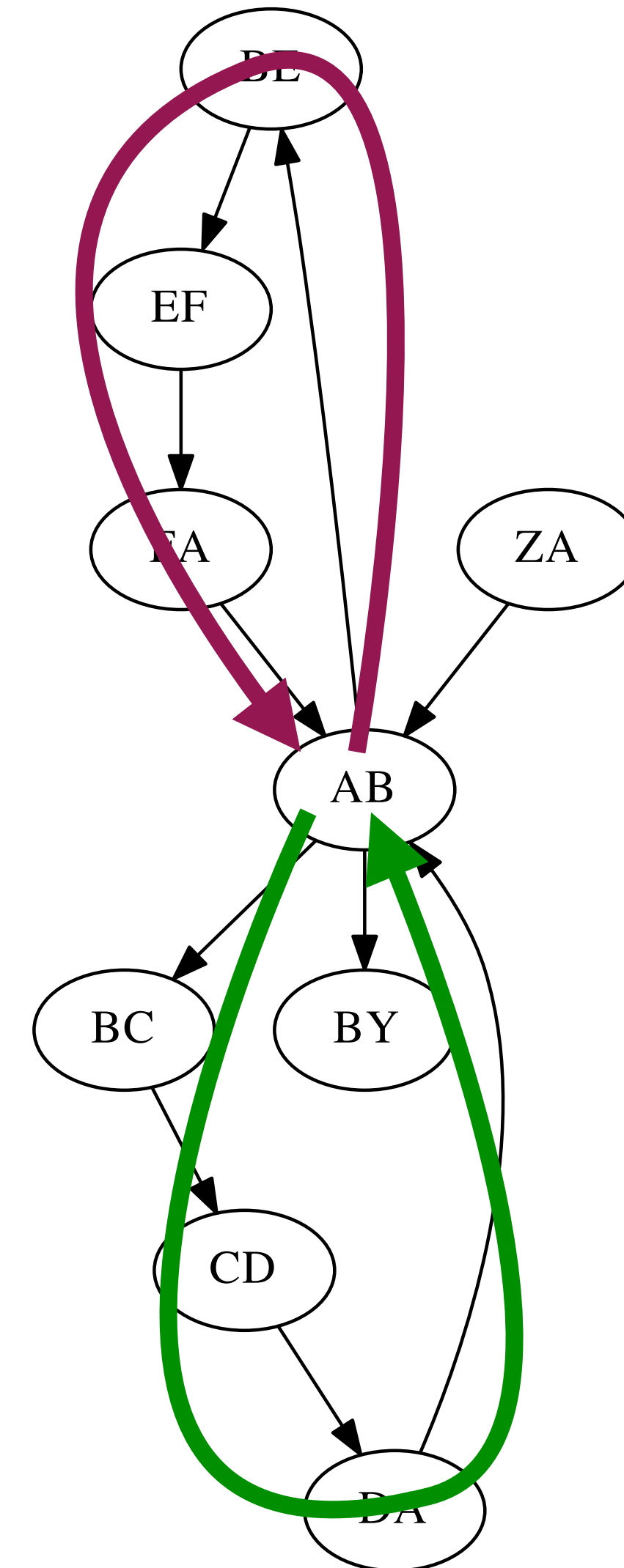
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We've been building DBGs assuming "perfect" sequencing: each k-mer reported exactly once, no mistakes. Real datasets aren't like that.

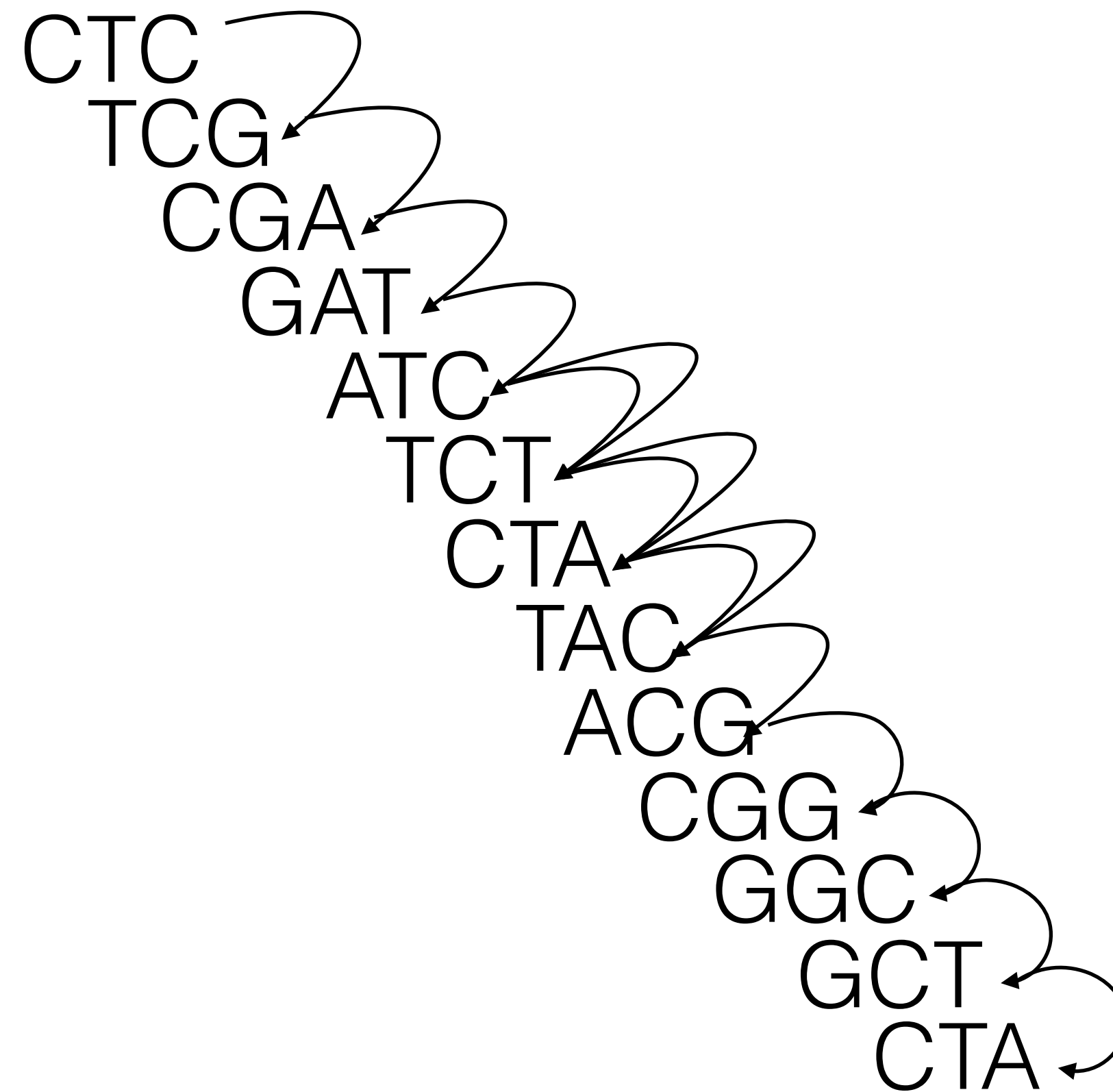


# Uneven coverage foils Eulerian Paths

r1: CTCGATCTAC

r2: ATCTACGGCTA

k=4

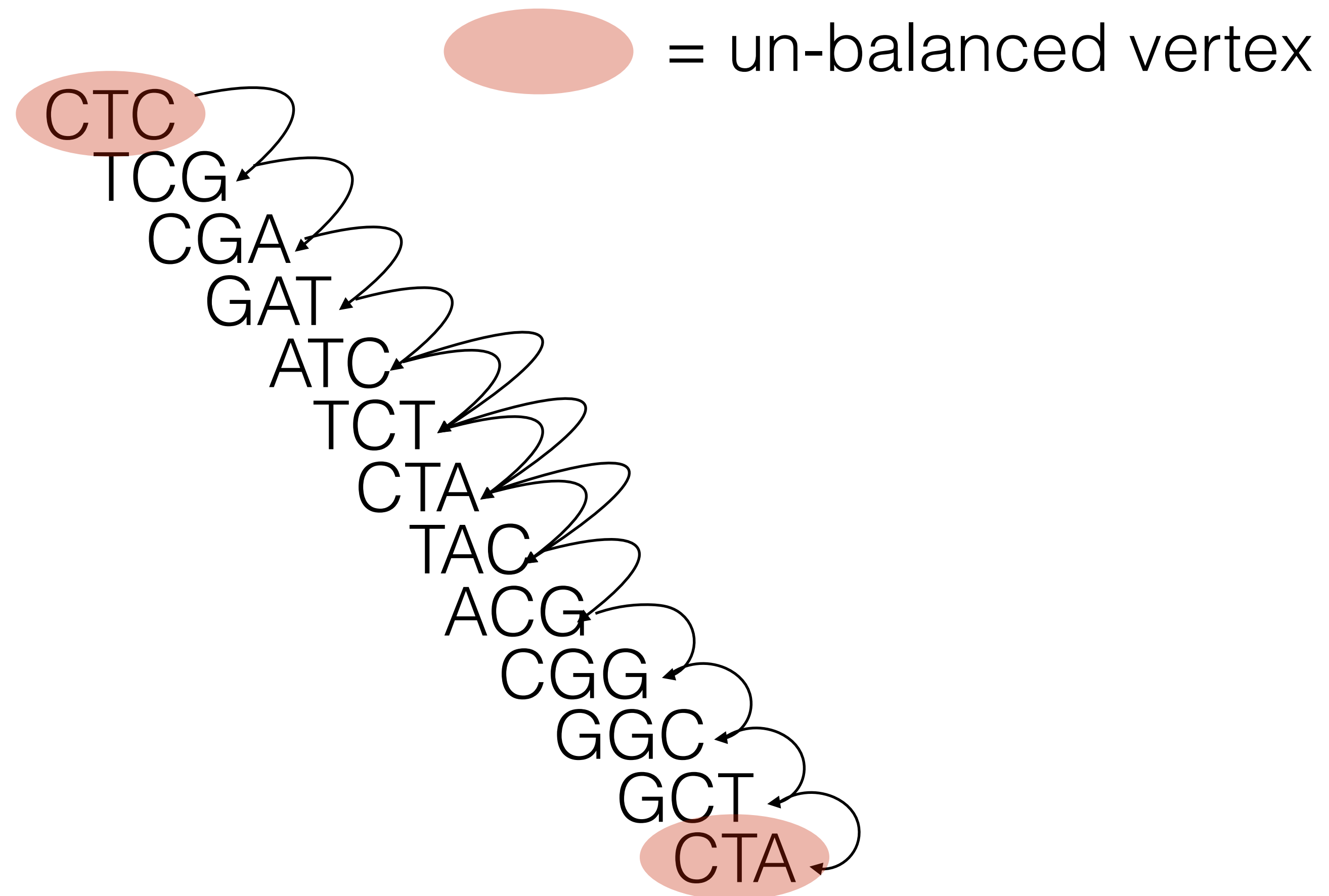


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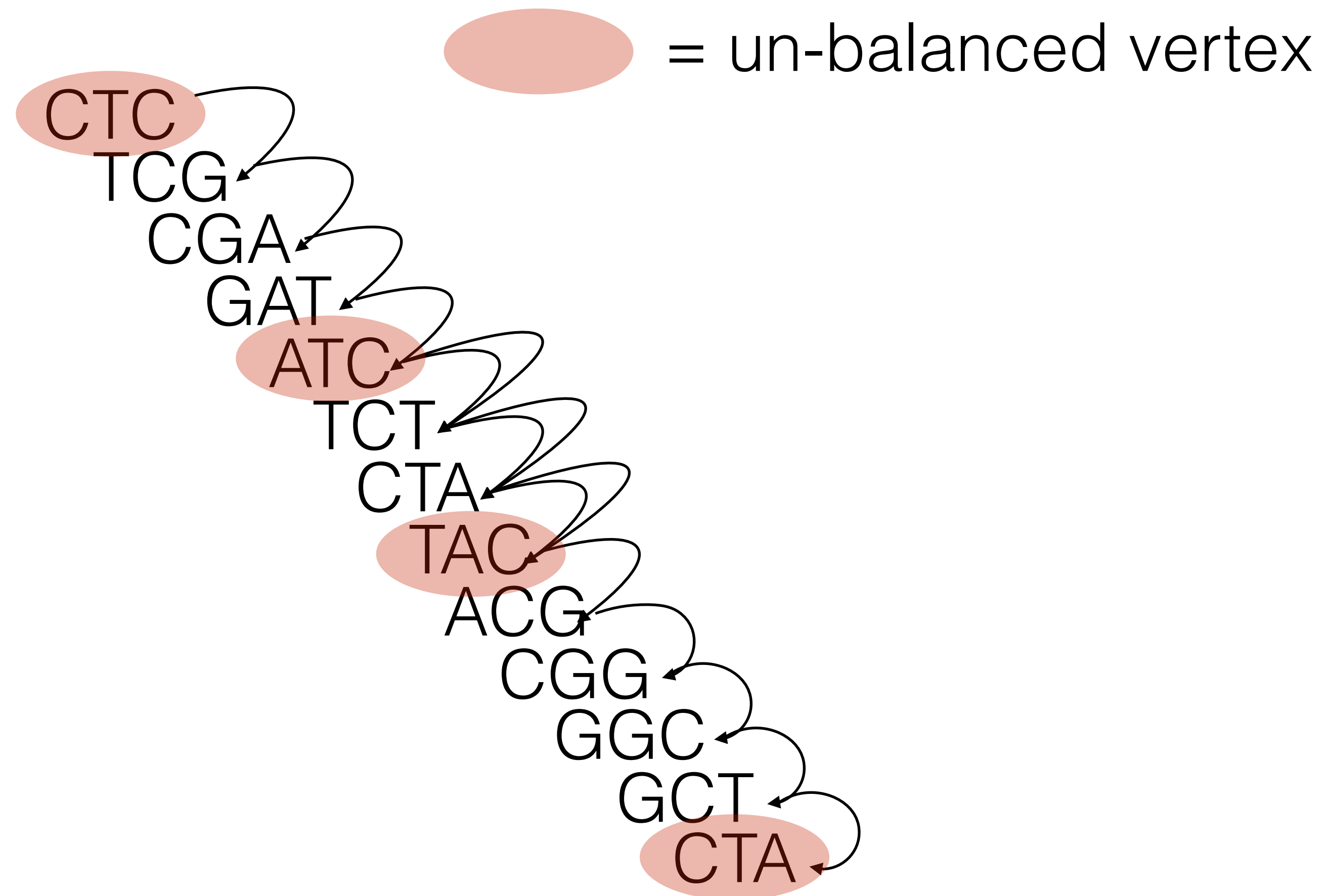


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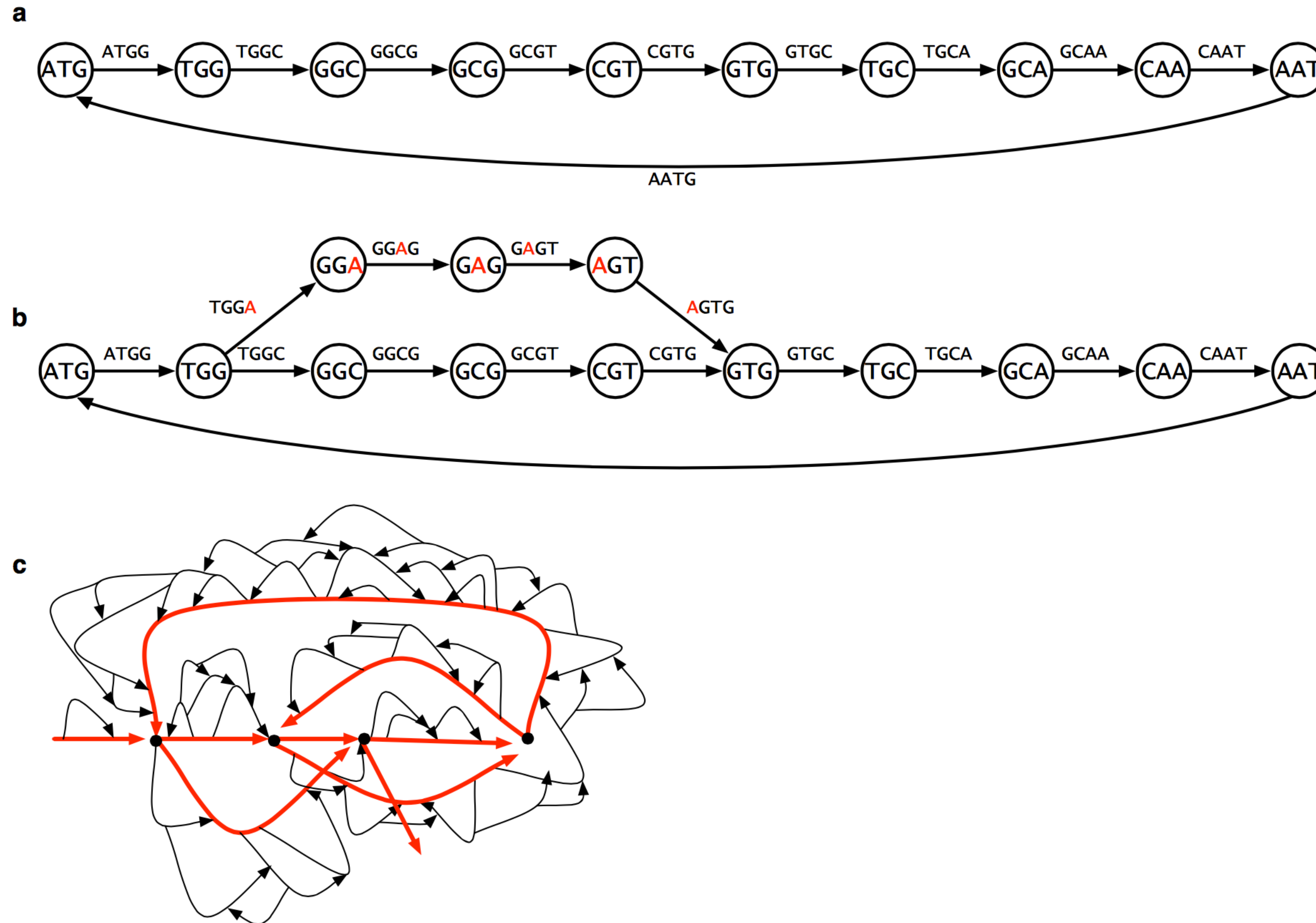
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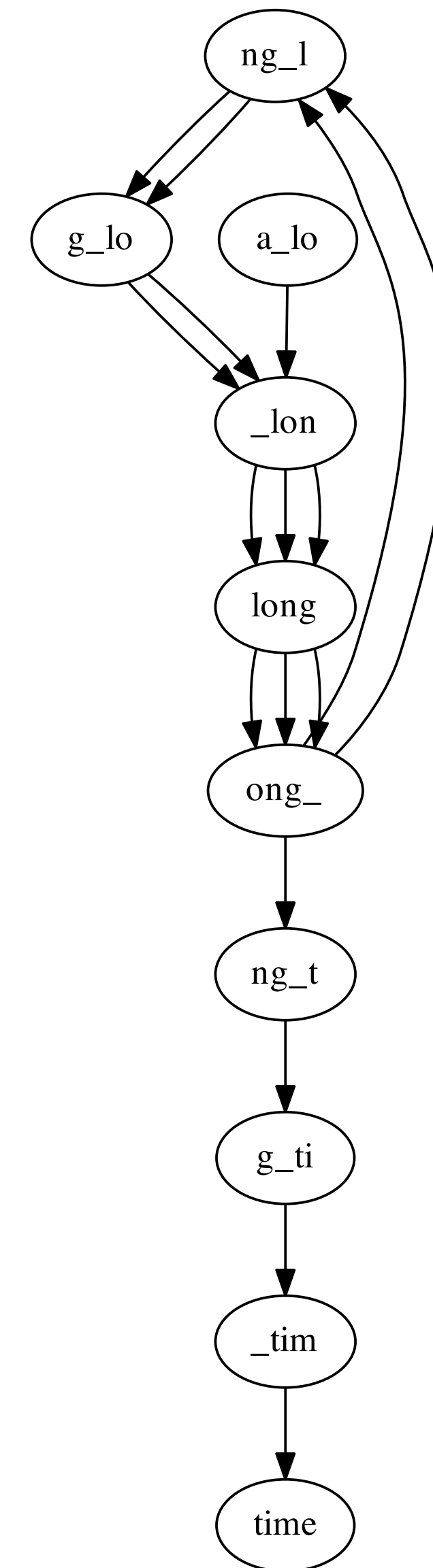
# Bursting bubbles



# De Bruijn graph

Gaps in coverage (missing k-mers) lead to disconnected or non-Eulerian graph

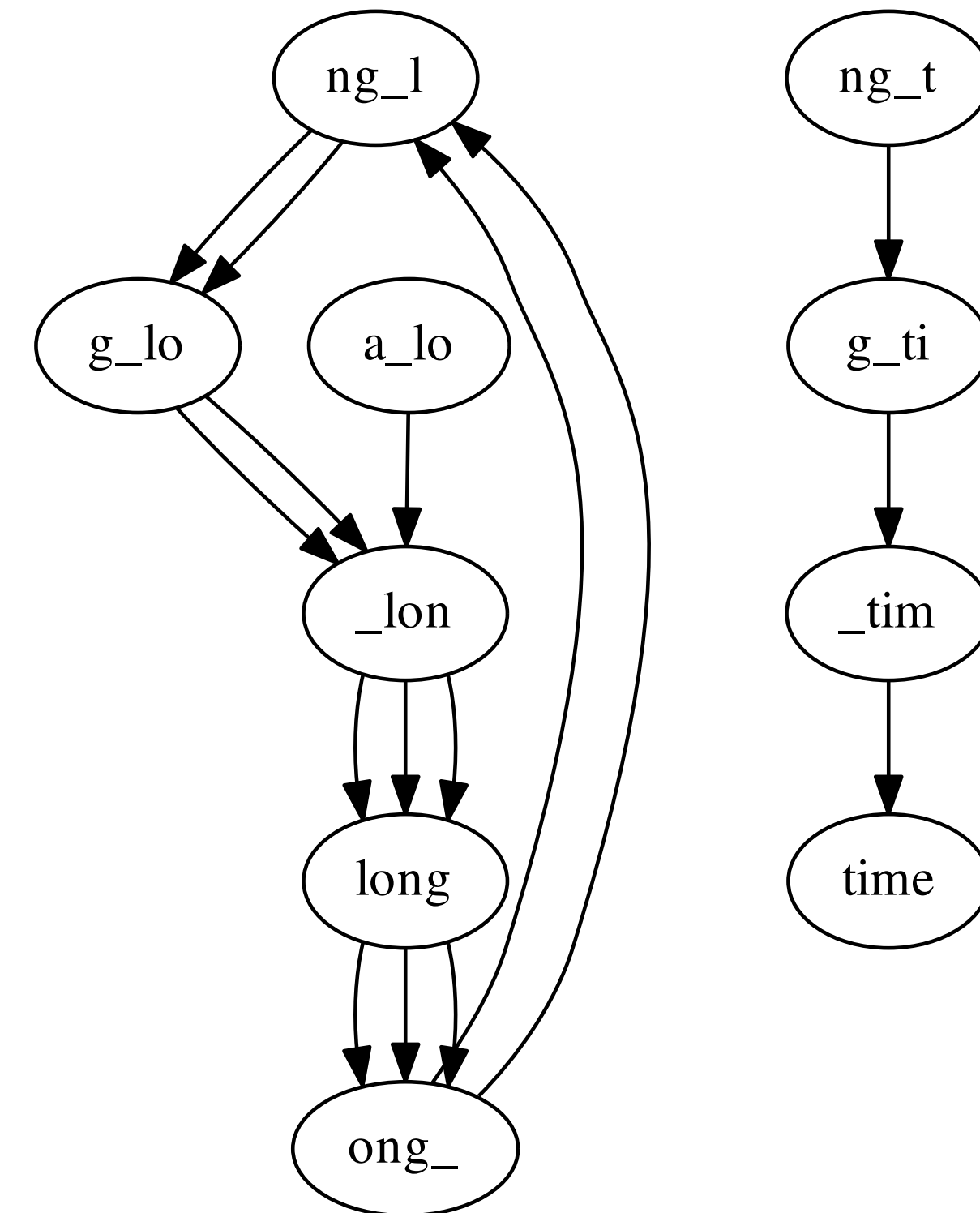
Graph for [a\\_long\\_long\\_long\\_time](#),  $k = 5$ :



# De Bruijn graph

Gaps in coverage (missing k-mers) lead to disconnected or non-Eulerian graph

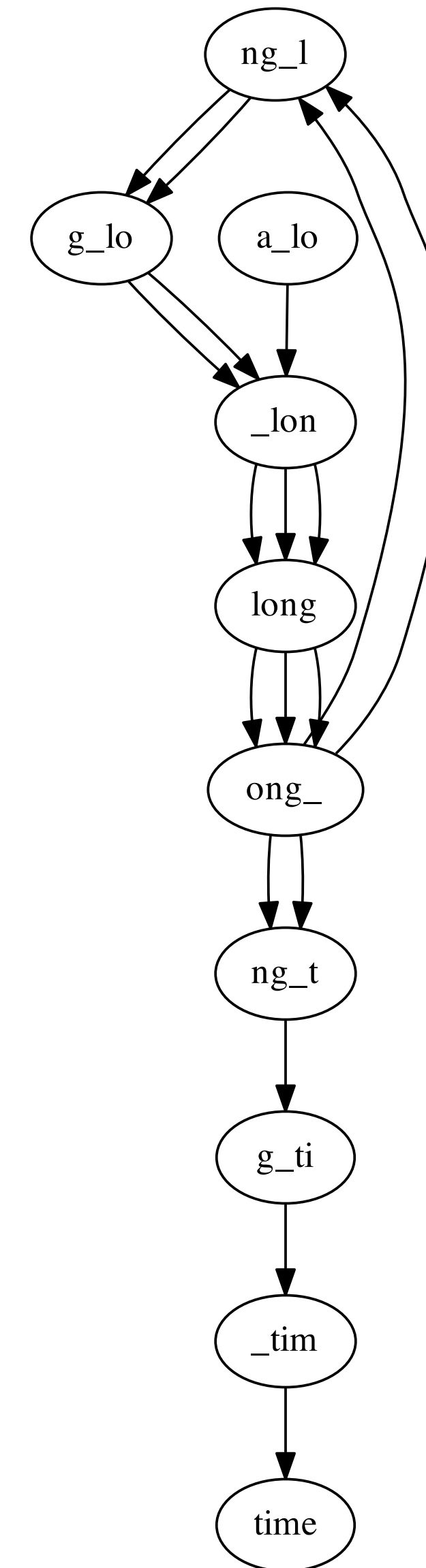
Graph for `a_long_long_long_time`,  $k = 5$  but omitting `ong_t`:



# De Bruijn graph

Coverage differences make graph non-Eulerian

Graph for `a_long_long_long_time`,  $k = 5$ , with extra copy of `ong_t`:

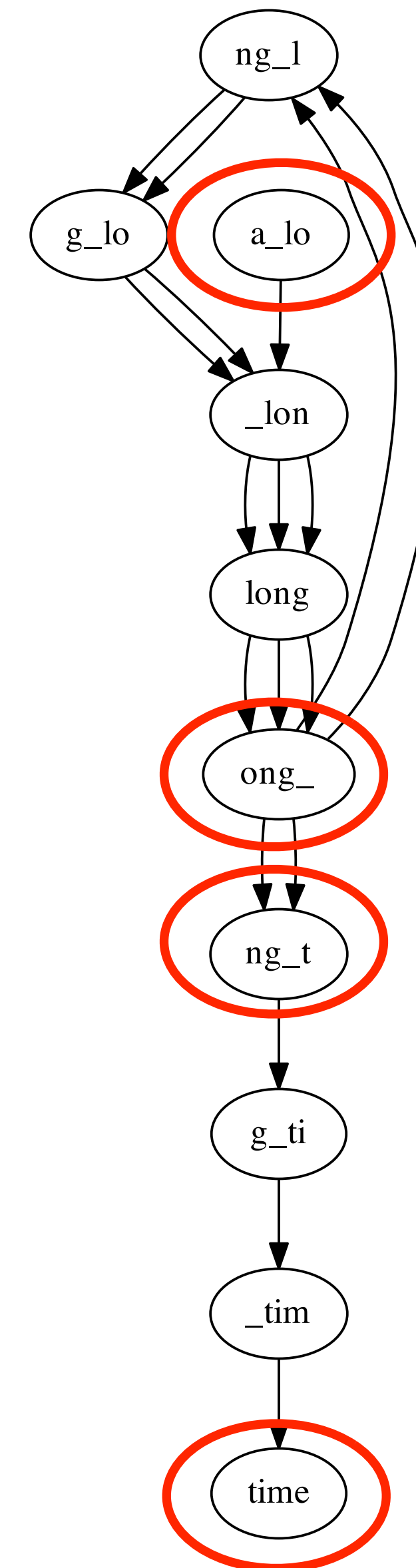


# De Bruijn graph

Coverage differences make graph non-Eulerian

Graph for `a_long_long_long_time`,  $k = 5$ , with extra copy of `ong_t`:

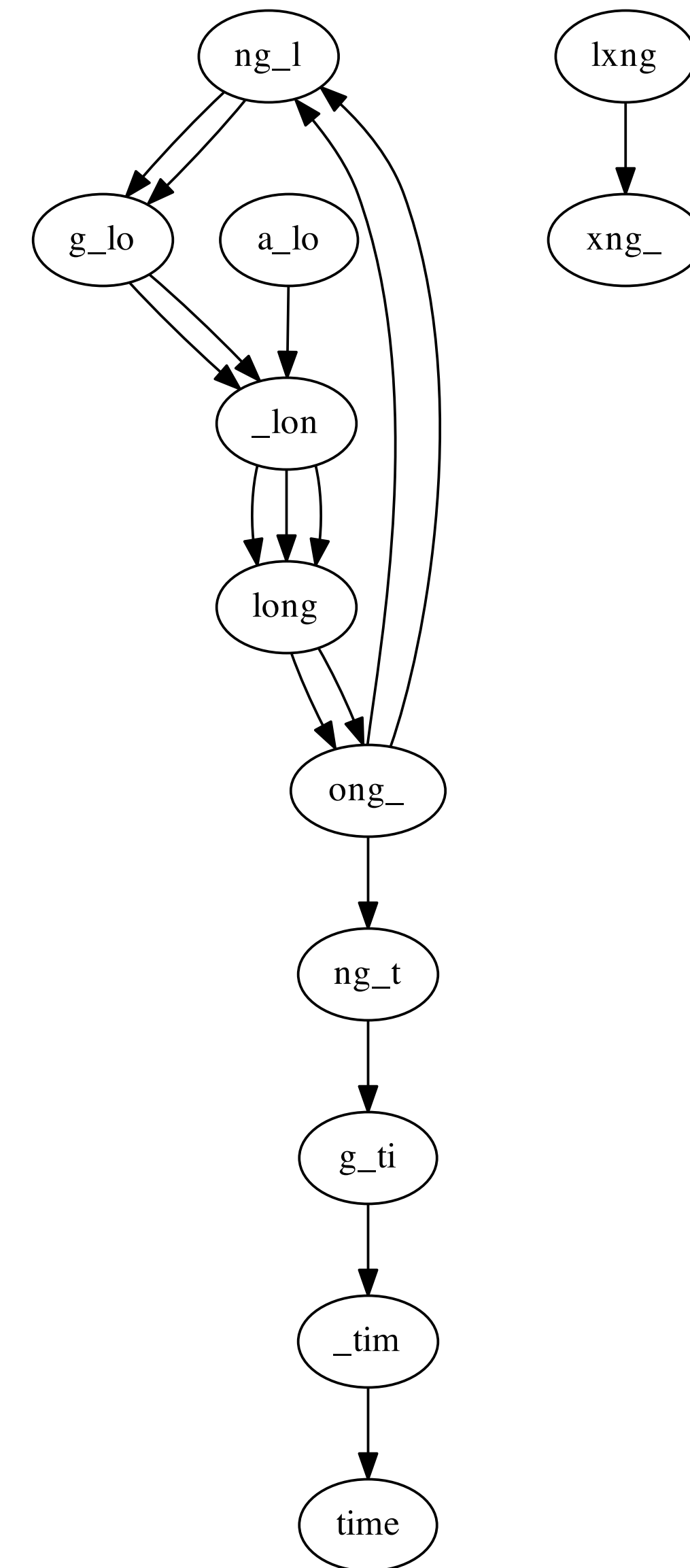
4 **semi-balanced** nodes



# De Bruijn graph

Errors and differences between chromosomes  
also lead to non-Eulerian graphs

Graph for `a_long_long_long_time`,  $k = 5$  but with  
error that turns one copy of `long_` into `lxng_`



# De Bruijn graph

Casting assembly as Eulerian walk is appealing, but not practical

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Uneven coverage, sequencing errors, etc make graph non-Eulerian

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Kingsford, Carl, Michael C. Schatz, and Mihai Pop. "Assembly complexity of prokaryotic genomes using short reads." BMC bioinformatics 11.1 (2010): 21.

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De Bruijn Superwalk Problem (DBSP) seeks a walk over the De Bruijn graph, where walk contains each read as a subwalk

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De Bruijn Superwalk Problem (DBSP) seeks a walk over the De Bruijn graph, where walk contains each read as a subwalk

Proven NP-hard!

Medvedev, Paul, et al. "Computability of models for sequence assembly." Algorithms in Bioinformatics. Springer Berlin Heidelberg, 2007. 289-301.

# De Bruijn graph

**In practice**, De Bruijn graph-based tools give up on unresolvable repeats and yield fragmented assemblies, just like OLC tools.

But first we note that using the De Bruijn graph representation has **other advantages...**

# De Bruijn graph

Say a sequencer produces  $d$  reads of length  $n$  from a genome of length  $m$

|                                        |                              |
|----------------------------------------|------------------------------|
| $d = 6 \times 10^9$ reads              | } $\approx 1$ sequencing run |
| $n = 100$ nt                           |                              |
| $m = 3 \times 10^9$ nt $\approx$ human |                              |

To build a De Bruijn graph in practice:

Pick  $k$ . Assume  $k \leq$  shortest read length ( $k = 30$  to  $50$  is common).

For each read:

For each  $k$ -mer:

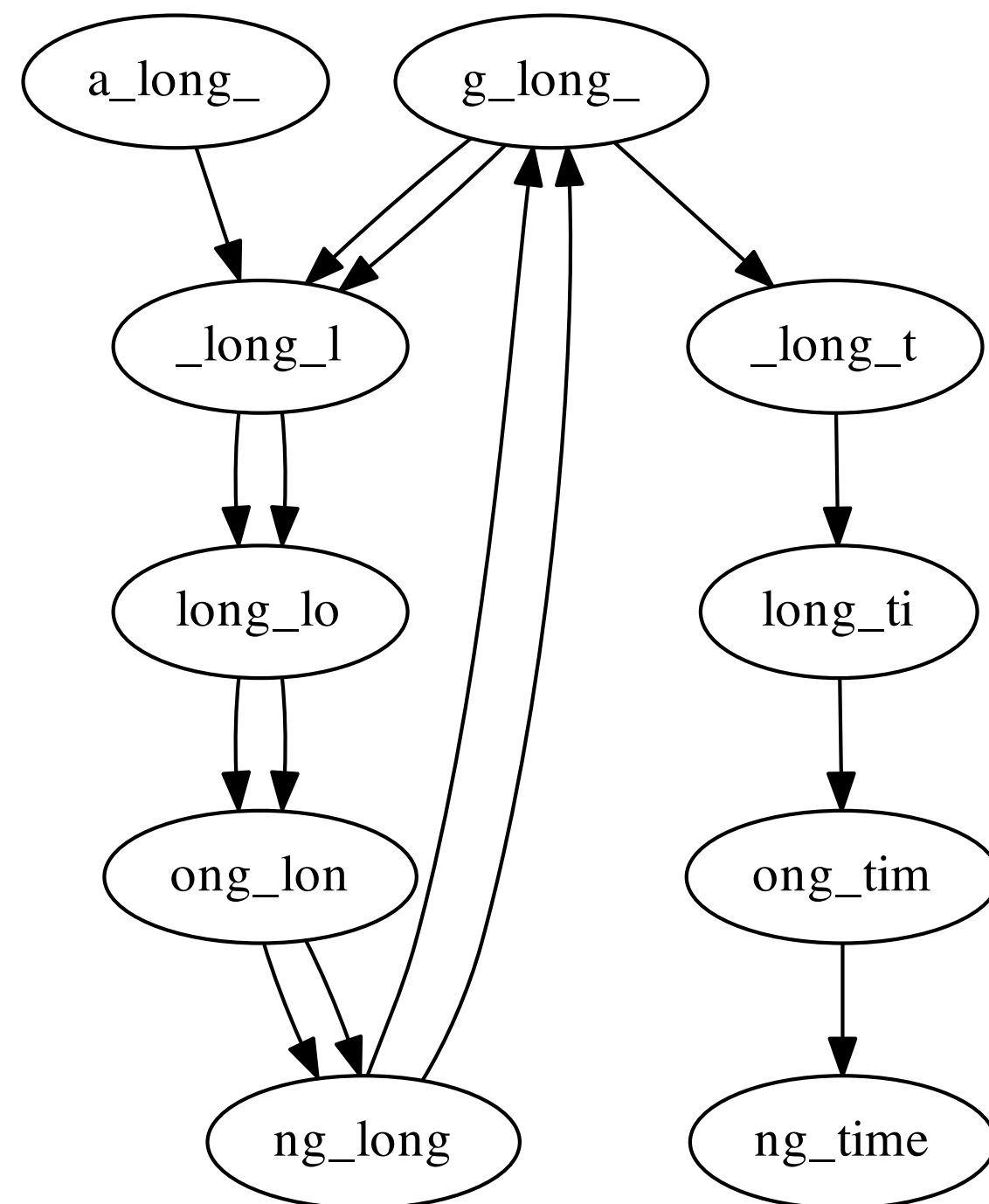
Add  $k$ -mer's left and right  $k-1$ -mers to graph if not there already. Draw an edge from left to right  $k-1$ -mer.

# De Bruijn graph

Pick  $k = 8$  Genome: `a_long_long_long_time`

Reads: `a_long_long_long`, `ng_long_l`, `g_long_time`

k-mers: `a_long_l` `ng_long` `g_long_t`  
`_long_lo` `g_long_l` `_long_ti`  
`-long_lon` `-long_tim`  
`ong_long` `ong_time`  
`ng_long`  
`g_long_l`  
`-long_lo`  
`-long_lon`  
`ong_long`



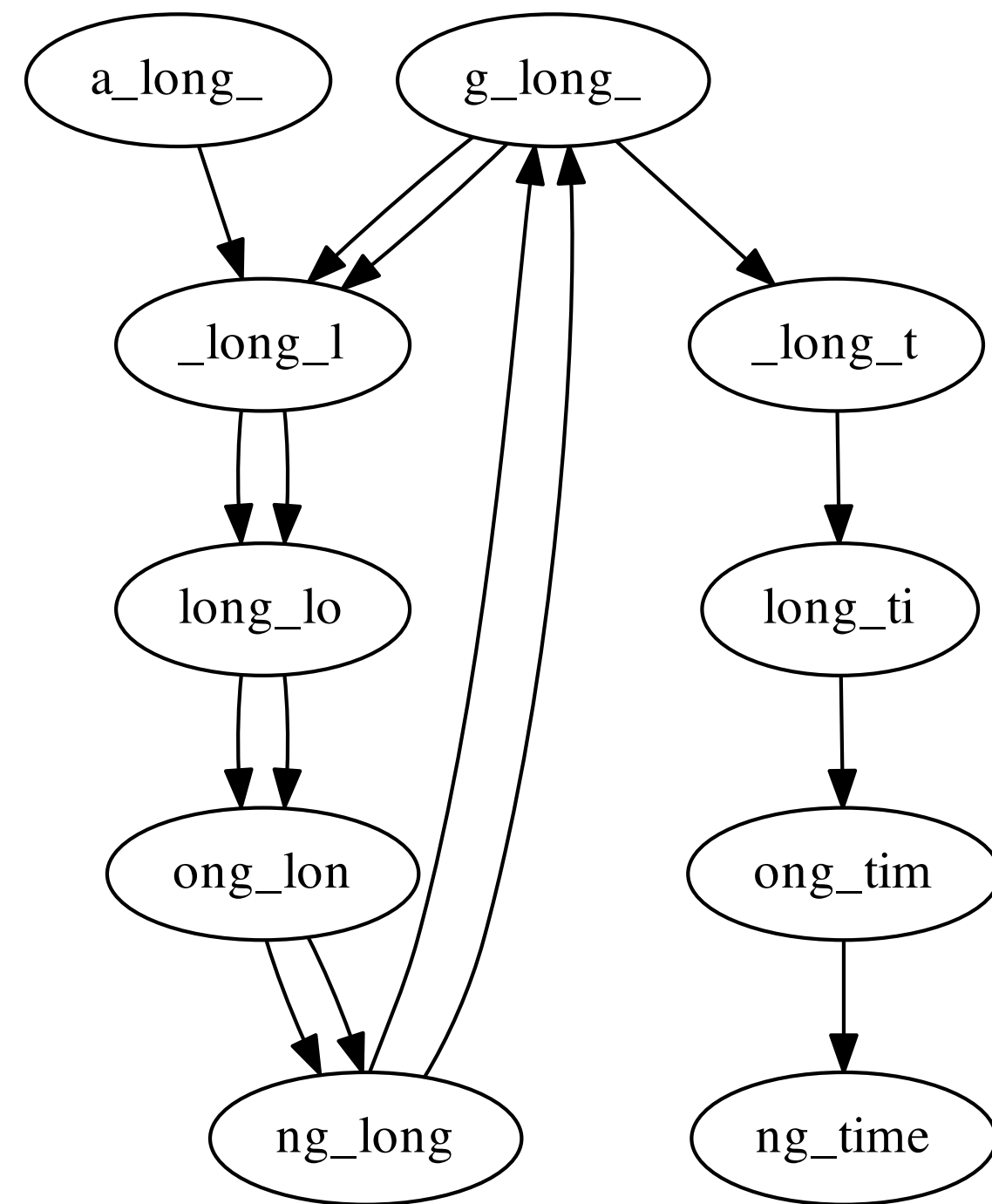
Given  $n$  (# reads),  $N$  (total length of all reads) and  $k$ , and assuming  $k < \text{length of shortest read}$ :

Exact number of k-mers:  $N - n(k-1)$   $O(N)$

This is also the number of edges,  $|E|$

Number of nodes  $|V|$  is at most  $2 \cdot |E|$ , but typically much smaller due to repeated  $k-1$ -mers

# De Bruijn graph



How much work to build graph?

For each  $k$ -mer, add 1 edge and up to 2 nodes

Reasonable to say this is  $O(1)$  expected work

Assume hash map encodes nodes & edges

Assume  $k-1$ -mers fit in  $O(1)$  machine words,  
and hashing  $O(1)$  machine words is  $O(1)$  work

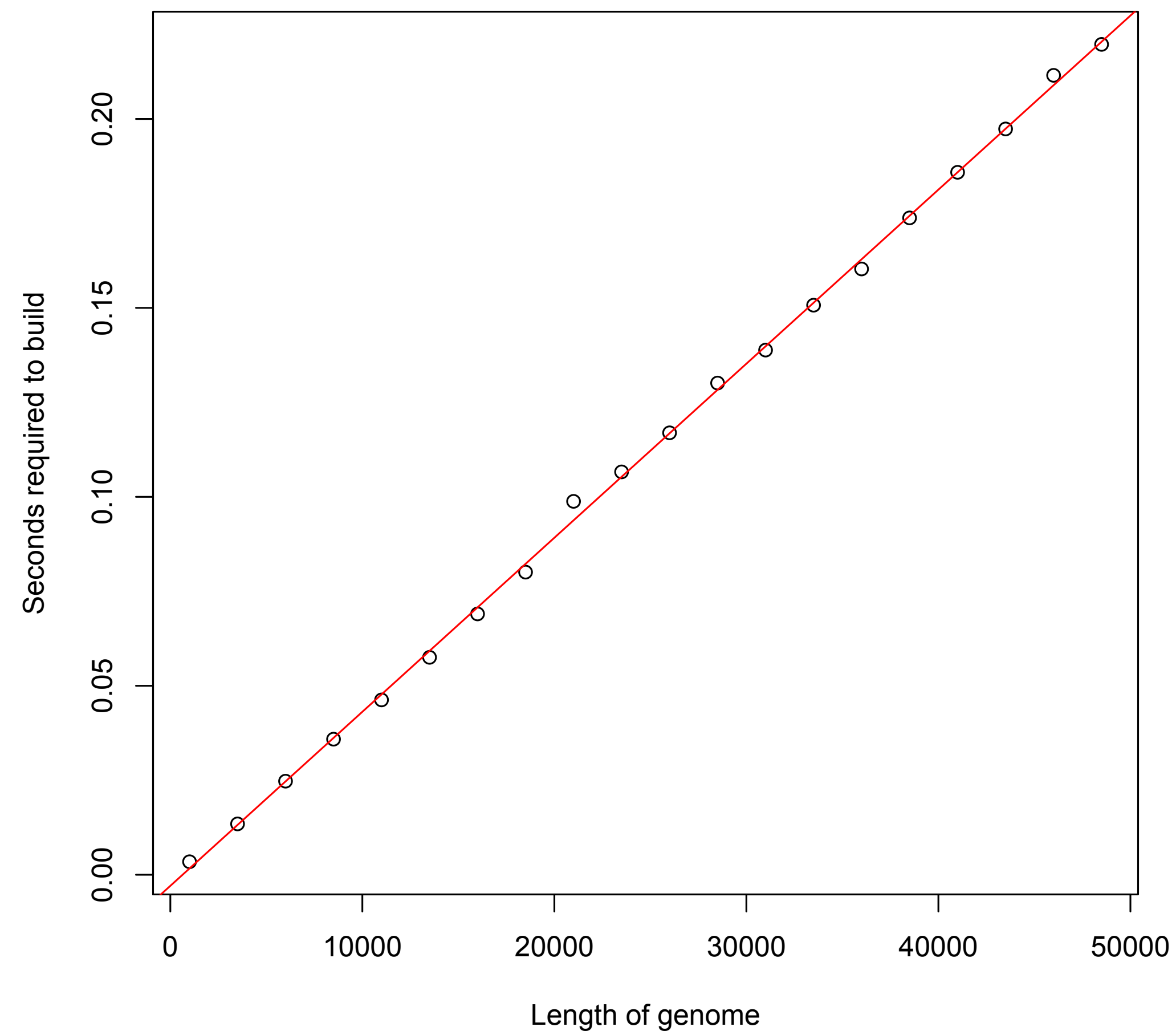
Querying / adding a key is  $O(1)$  expected work

$O(1)$  expected work for 1  $k$ -mer,  $O(N)$  overall

# De Bruijn graph

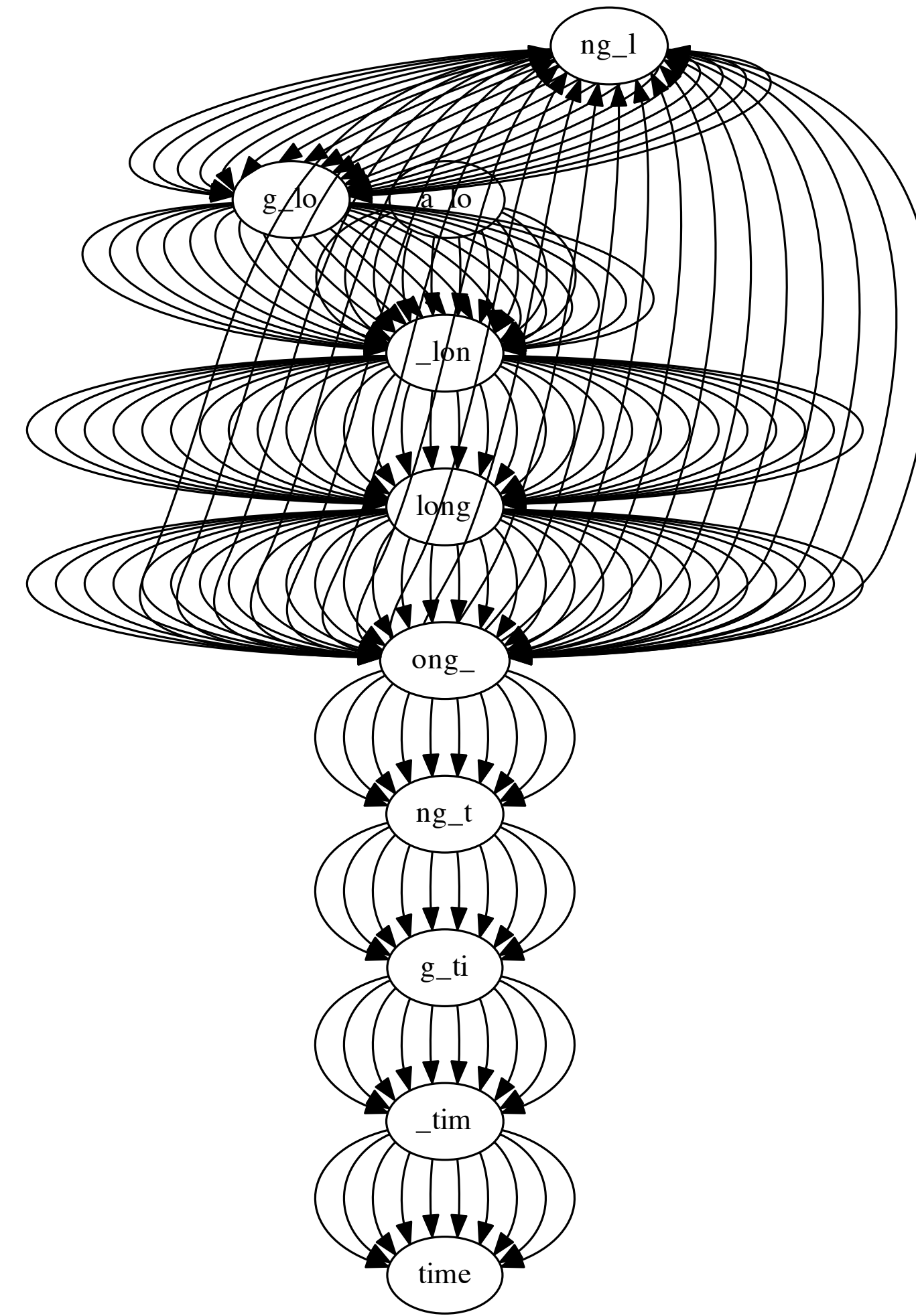
Timed De Bruijn graph construction applied to progressively longer prefixes of lambda phage genome,  $k = 14$

$O(N)$  expectation  
appears to work in  
practice, at least for this  
small example



# De Bruijn graph

In typical assembly projects,  
average coverage is  $\sim 30 - 50$



# De Bruijn graph

Recall *average coverage*: average # reads covering a genome position

|                                   |                 |
|-----------------------------------|-----------------|
| CTAGGCCCTCAATTTT                  |                 |
| CTCTAGGCCCTCAATTTT                |                 |
| GGCTCTAGGCCCTCATTTT               |                 |
| CTCGGCTCTAGCCCCTCATTT             |                 |
| TATCTCGACTCTAGGCCCTCA             | 177 nucleotides |
| TATCTCGACTCTAGGCC                 |                 |
| TCTATATCTCGGCTCTAGG               |                 |
| GGCGTCTATATCTCG                   |                 |
| GGCGTCGATATCT                     |                 |
| GGCGTCTATATCT                     |                 |
| GGCGTCTATATCTCGGCTCTAGGCCCTCATTTT | 35 nucleotides  |

$$\text{Average coverage} = 177 / 35 \approx 7x$$

# De Bruijn graph

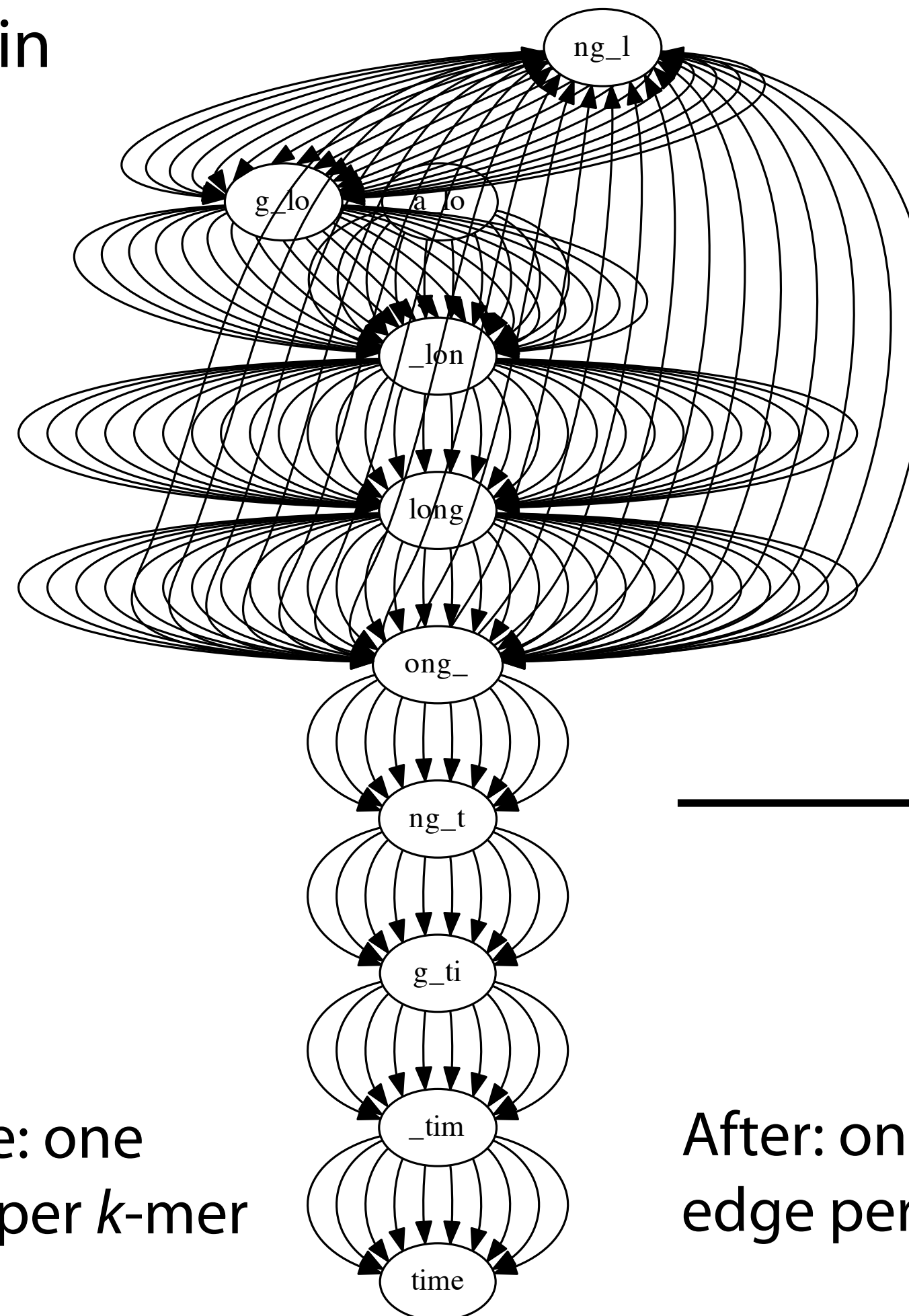
In typical assembly projects, average coverage is  $\sim 30 - 50$

Same edge might appear in dozens of copies; let's use edge *weights* instead

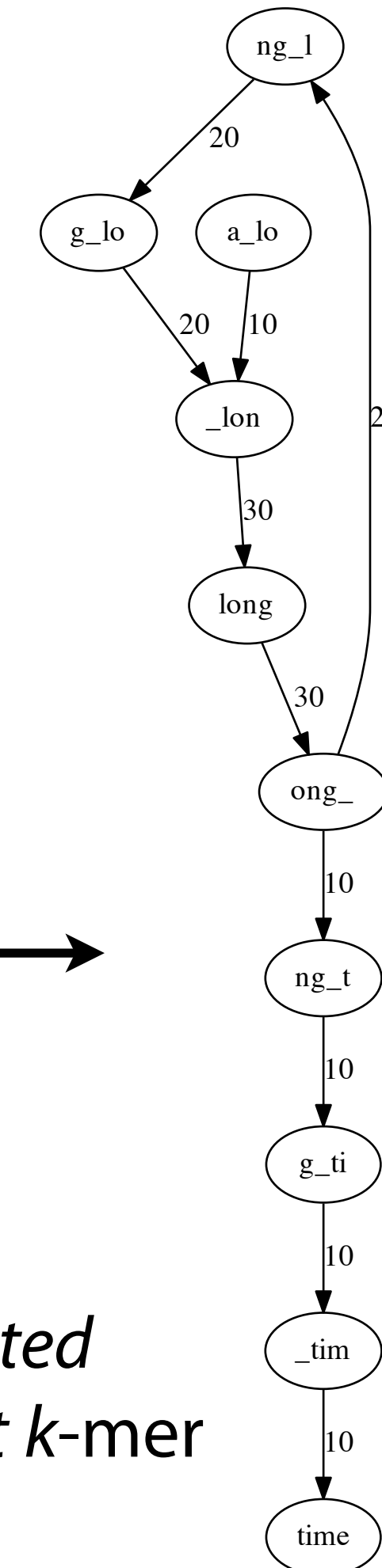
Weight = # times  $k$ -mer occurs

Using weights, there's one *weighted* edge for each *distinct*  $k$ -mer

Before: one edge per  $k$ -mer



After: one *weighted* edge per *distinct*  $k$ -mer



# De Bruijn graph

# of nodes and edges both  $O(N)$ ;  $N$  is total length of all reads

Say (a) reads are error-free, (b) we have one *weighted* edge for each *distinct*  $k$ -mer, and (c) length of genome is  $G$

There's one node for each distinct  $k-1$ -mer, one edge for each distinct  $k$ -mer

Can't be more distinct  $k$ -mers than there are  $k$ -mers in the genome; likewise for  $k-1$ -mers

So # of nodes and edges are also both  $O(G)$

Combine with the  $O(N)$  bound and the # of nodes and edges are both  $O(\min(N, G))$

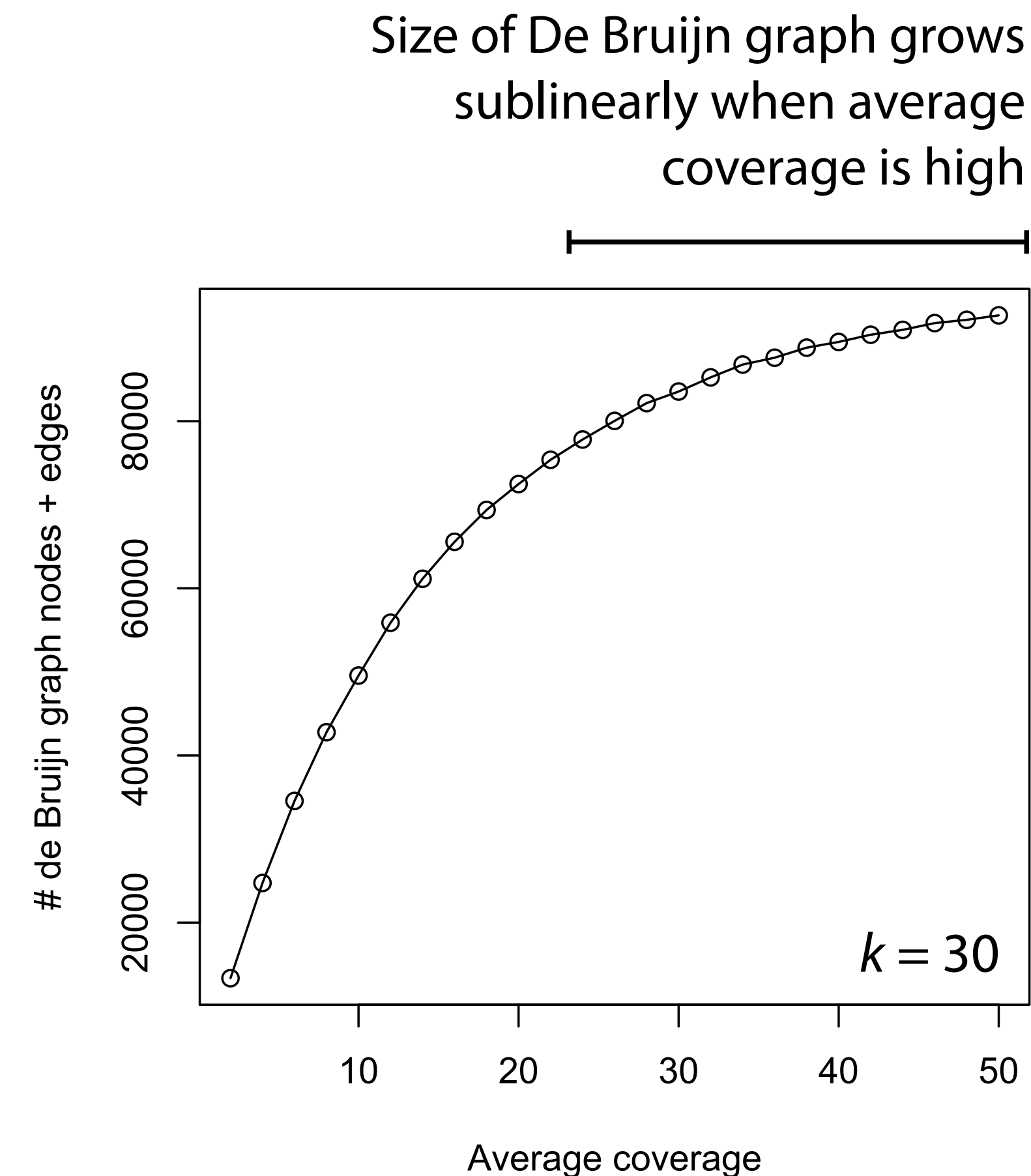
# De Bruijn graph

With high average coverage,  $O(G)$  size bound is advantageous

Genome = lambda phage (~ 48.5 K nt)

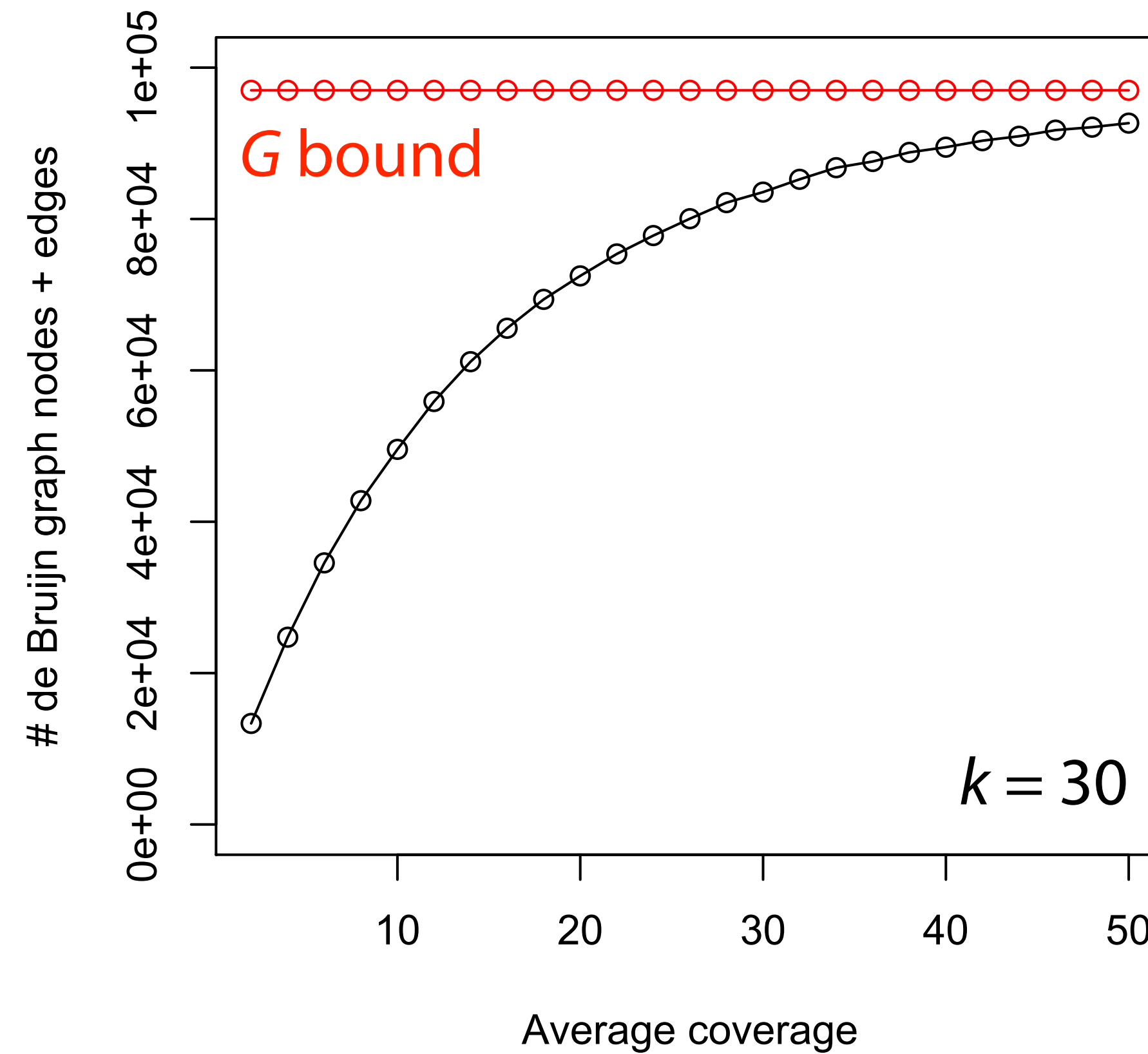
Draw random  $k$ -mers until target average coverage is reached (x axis)

Build De Bruijn graph and total the # of nodes and edges (y axis)



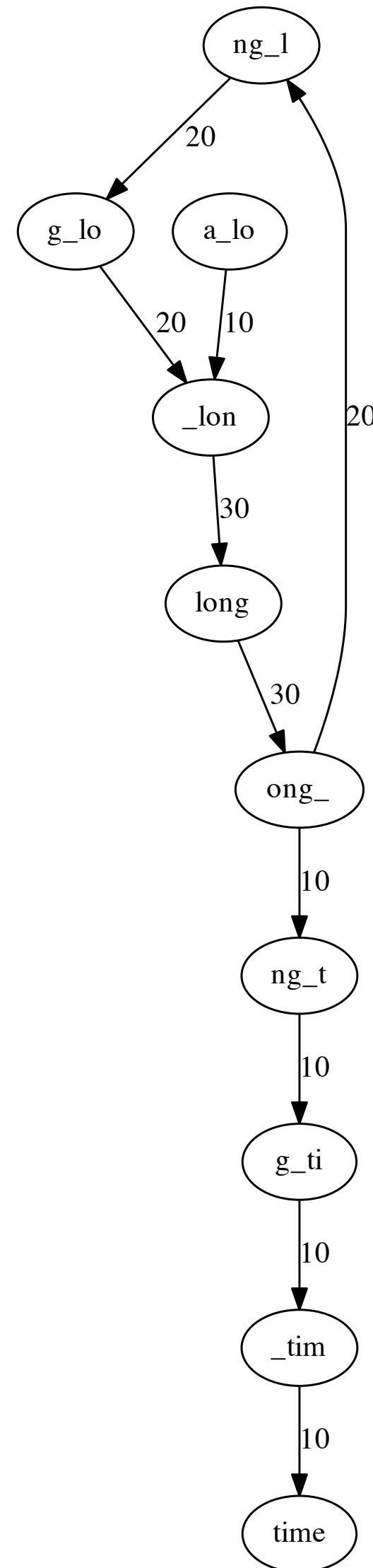
# Error correction

When data is error-free, # nodes, edges in de Bruijn graph is  $O(\min(G, N))$



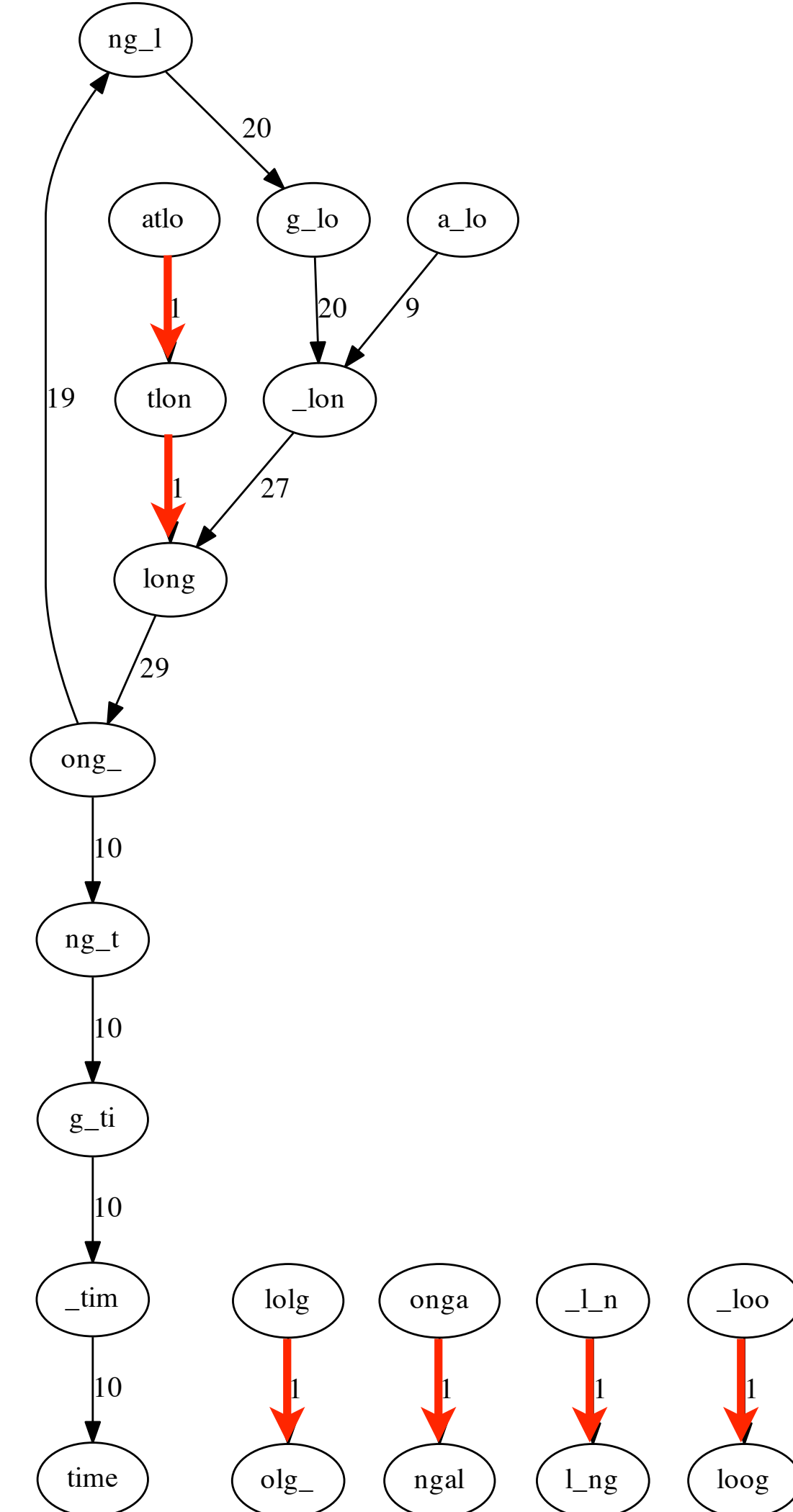
What about data with sequencing errors?

# Error correction



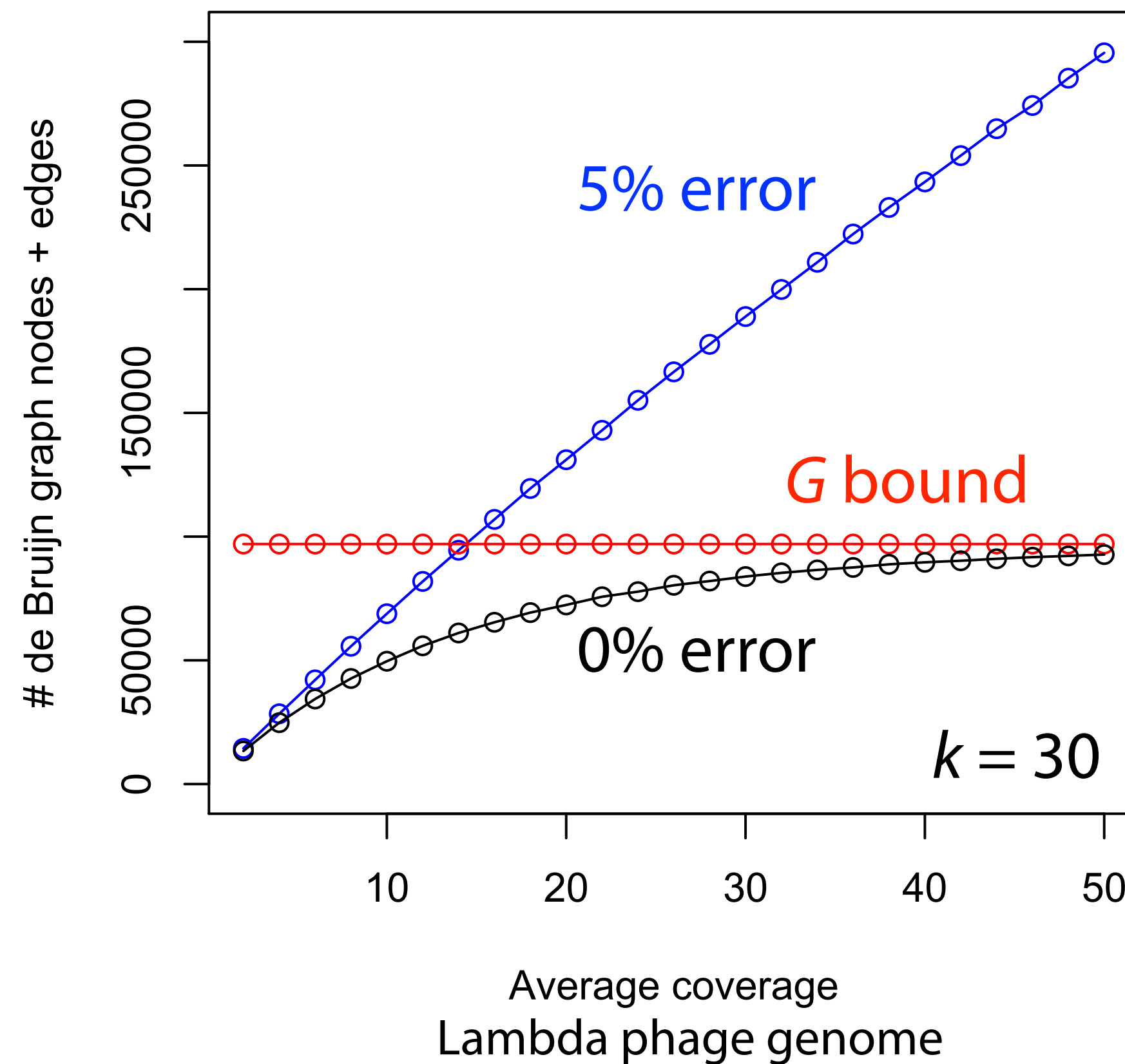
Take an example we saw (left) and mutate a  $k$ -mer character to a random other character with probability 1% (right)

6 errors result in 10 new nodes and **6 new weighted edges**, all with weight 1



# Error correction

As more  $k$ -mers overlap errors, # nodes, edges approach  $N$

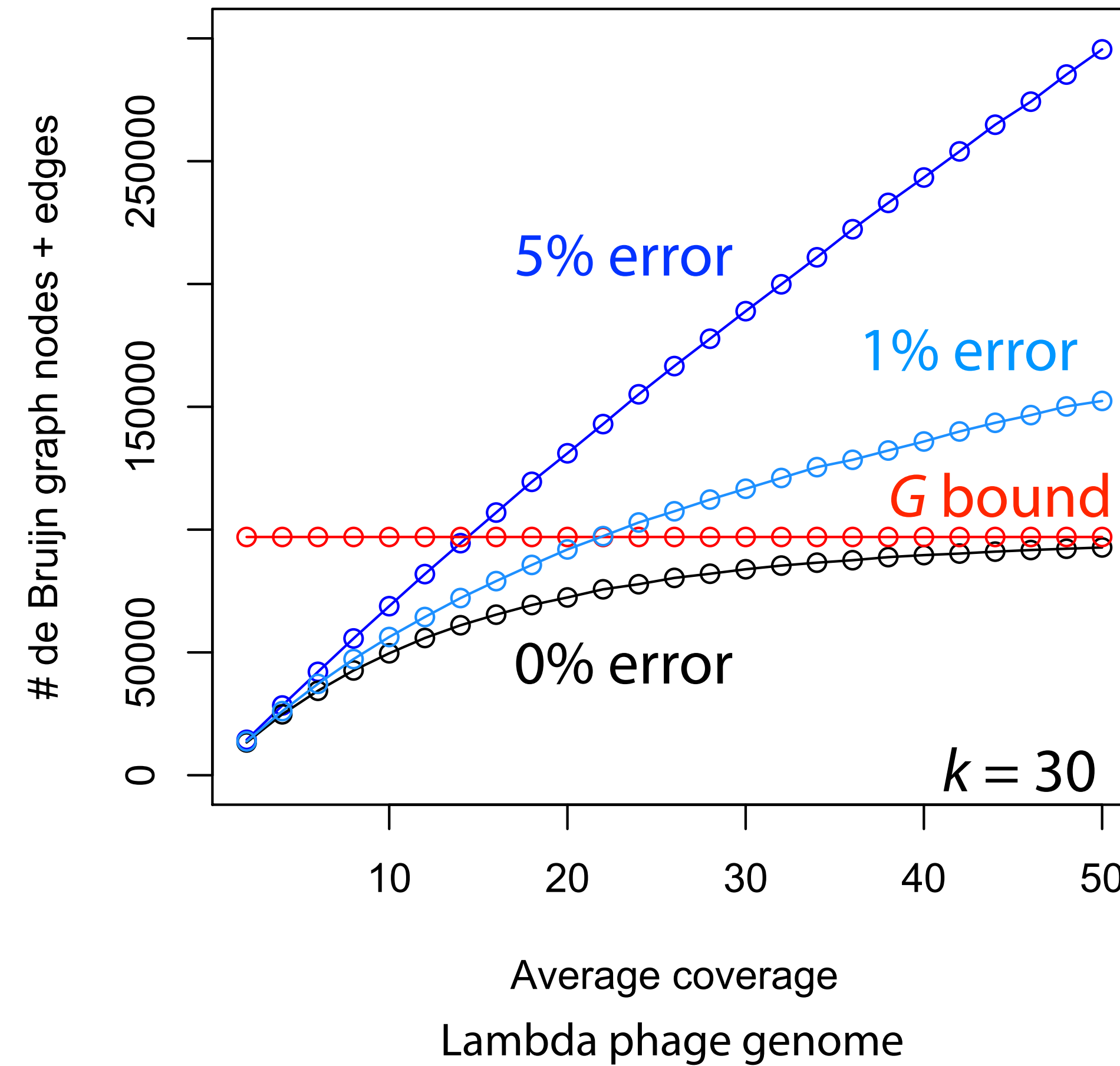


Same experiment as before but with 5% error added

Errors wipe out much of the benefit of the  $G$  bound

Instead of  $O(\min(G, N))$ , we have something more like  $O(N)$

# Error correction



# Error correction

If we can correct sequencing errors up-front, we can prevent De Bruijn graph from growing much beyond the  $G$  bound

How do we correct errors?

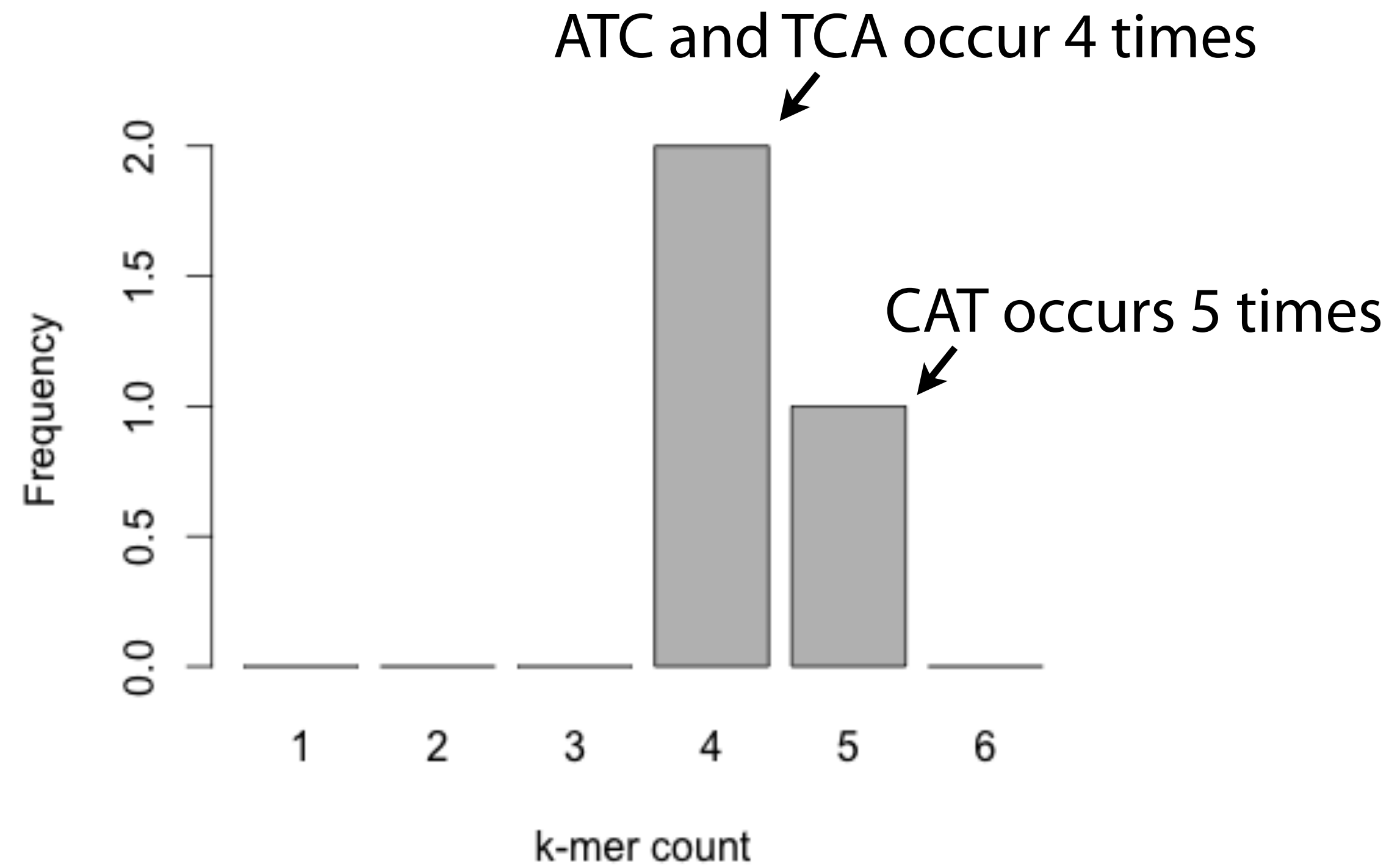
Analogy: design a spell checker for a language you've never seen before. How do you come up with suggestions?

# Error correction

$k$ -mer count histogram:

x axis is an integer  $k$ -mer count, y axis is # distinct  $k$ -mers with that count

Right: such a histogram for 3-mers of CATCATCATCATCAT:



# Error correction

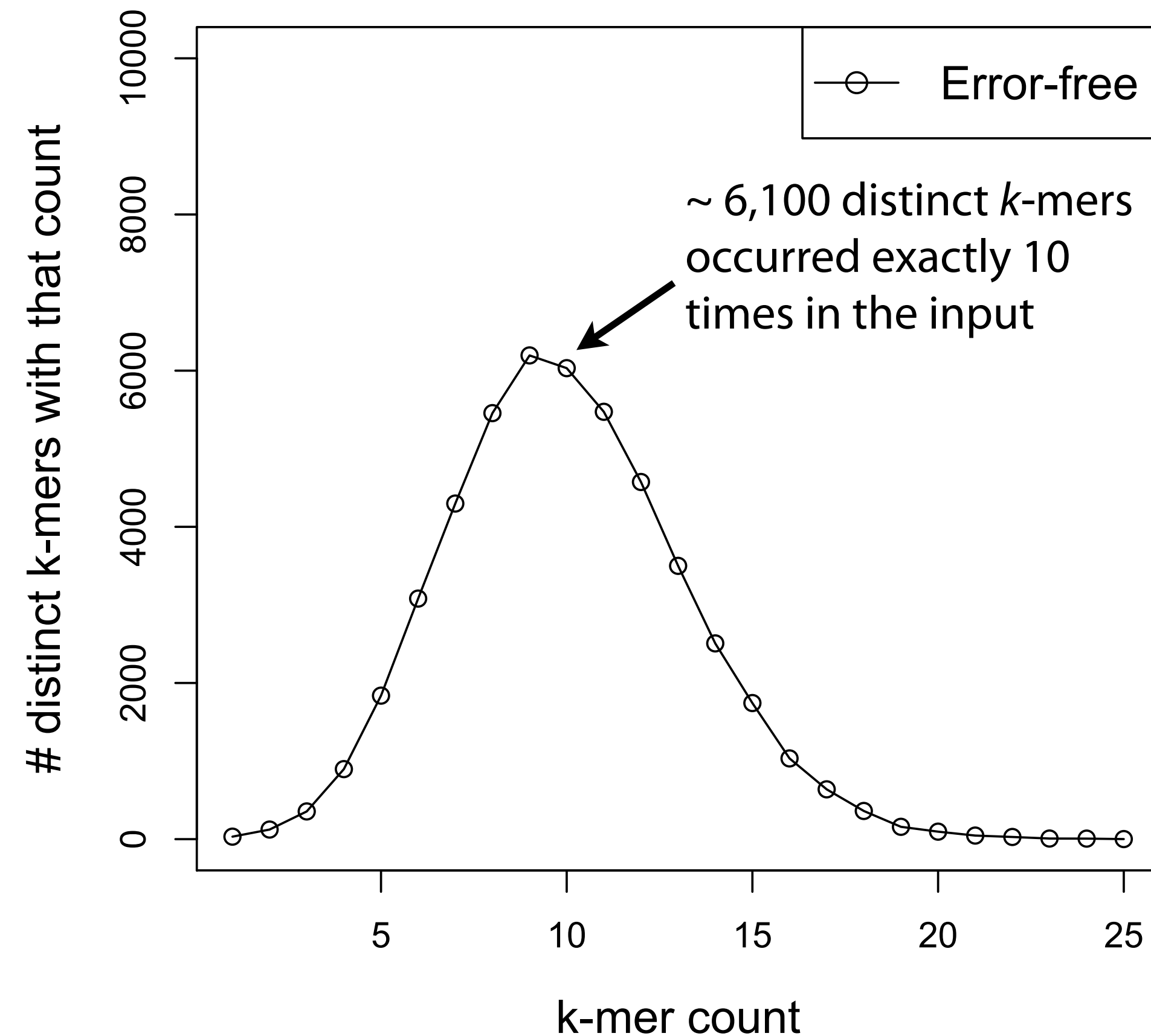
Say we have error-free sequencing reads drawn from a genome.

The amount of sequencing is such that average coverage = 200.

Let  $k = 20$

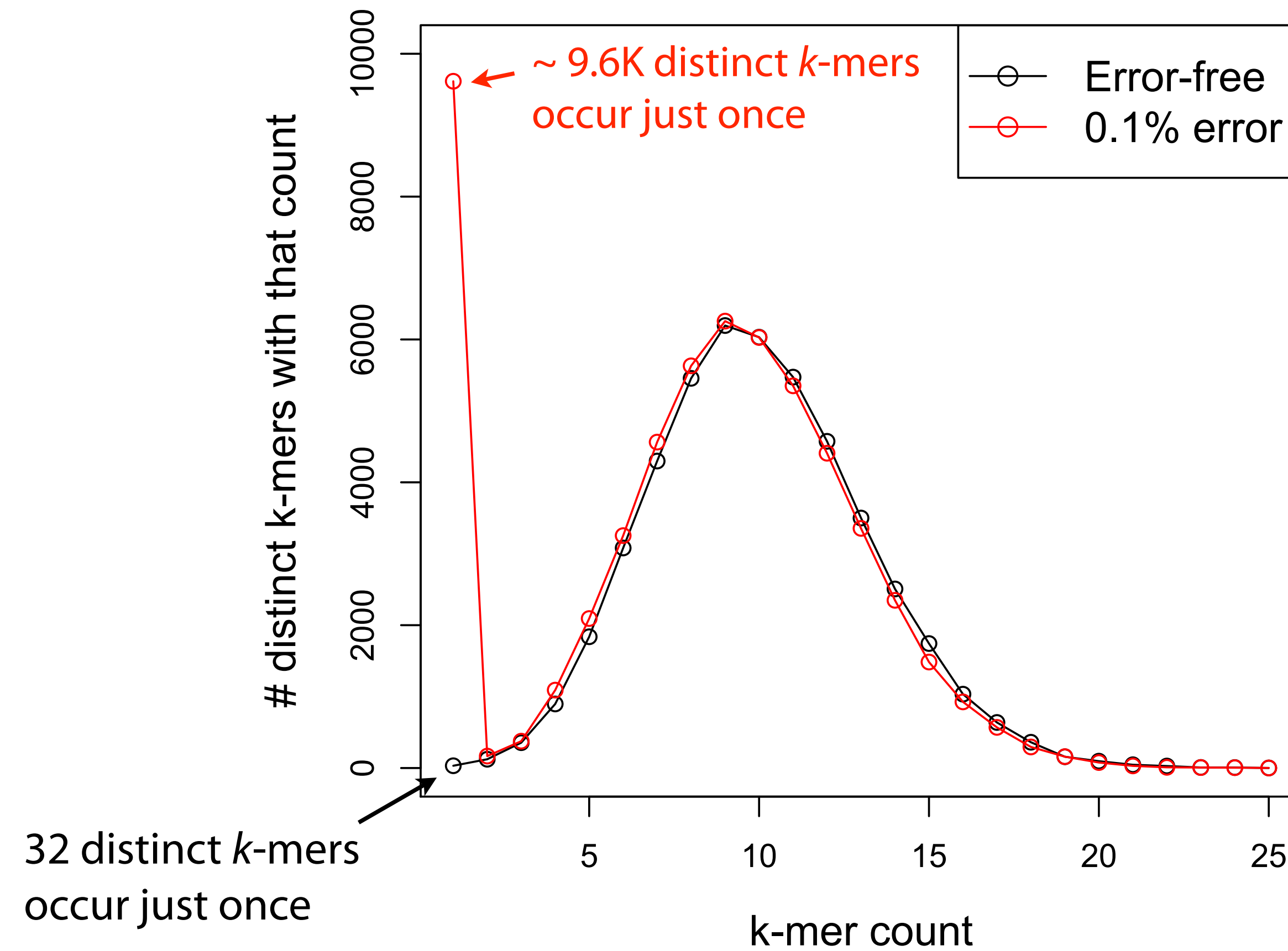
How would the picture change for data with 1% error rate?

Hint: errors usually change high-count  $k$ -mer into low-count  $k$ -mer



# Error correction

$k$ -mers with errors usually occur fewer times than error-free  $k$ -mers

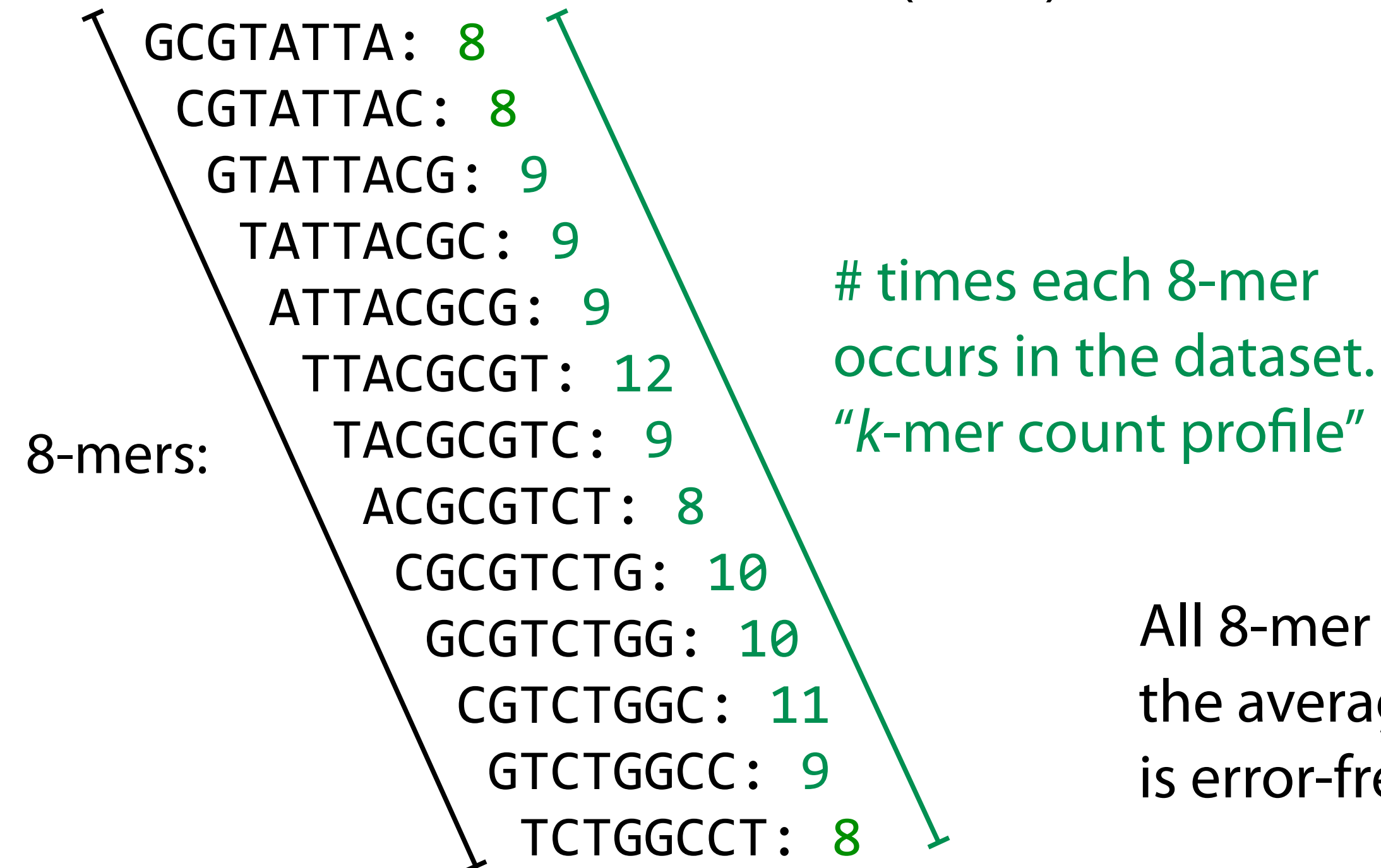


# Error correction

Idea: errors tend to turn frequent  $k$ -mers to infrequent  $k$ -mers, so corrections should do the reverse

Say we have a collection of reads where each distinct 8-mer occurs an average of  $\sim 10$  times, and we have the following read:

Read: GCGTATTACGCGTCTGGCCT (20 nt)

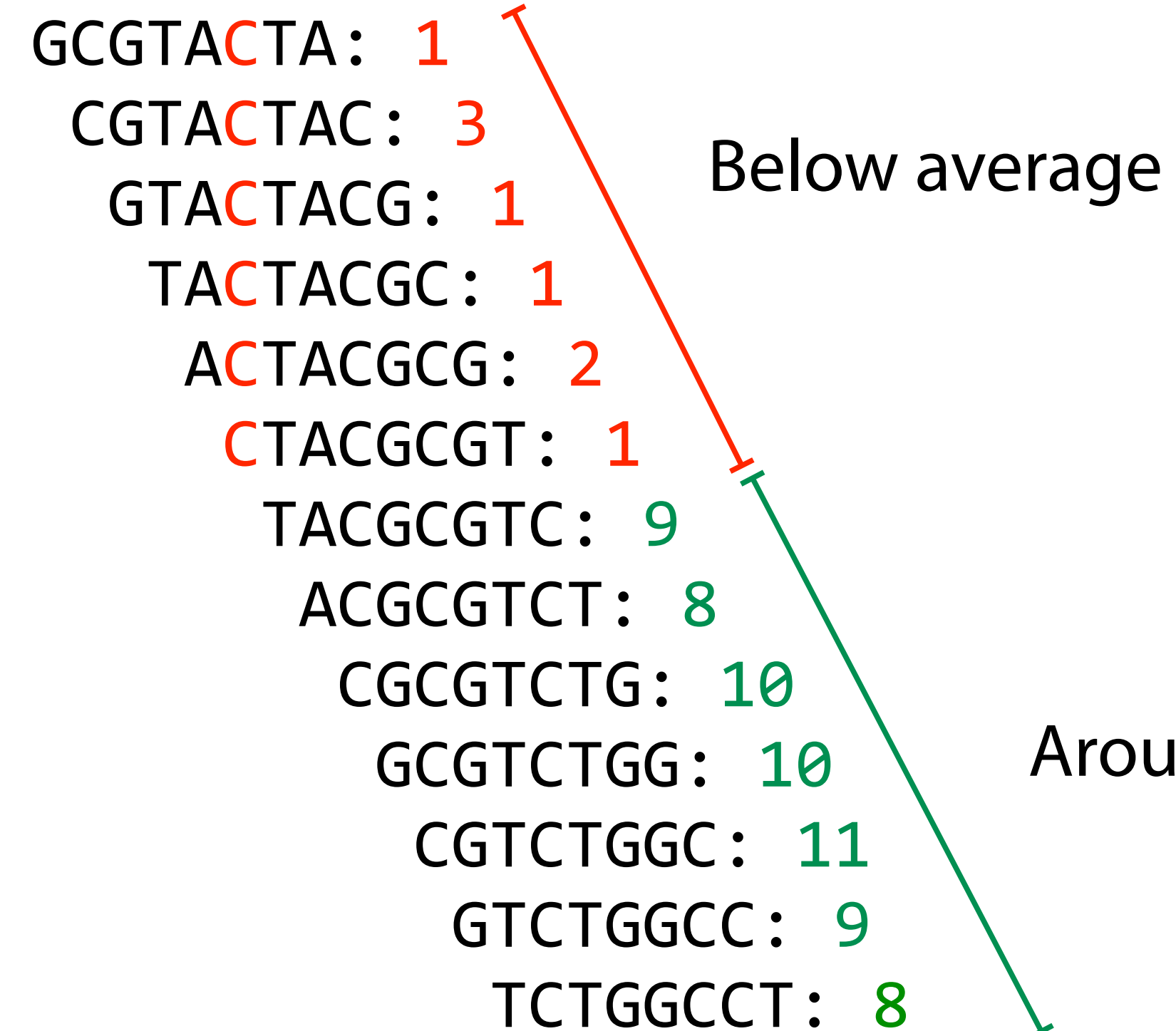


All 8-mer counts are around the average, suggesting read is error-free

# Error correction

Suppose there's an **error**

Read: GCGTAC**T**ACGCGTCTGGCCT



*k*-mer count profile has corresponding stretch of below-average counts

# Error correction

*k*-mer count profiles when errors are in different parts of the read:

GCGTAC**T**ACGCGTCTGGCCT

GCGTAC**T**A: 1

CGTAC**T**AC: 3

GTAC**T**ACG: 1

TAC**T**ACGC: 1

A**T**ACGCG: 2

**C**TACGCGT: 1

TACGCGTC: 9

ACGCGTCT: 8

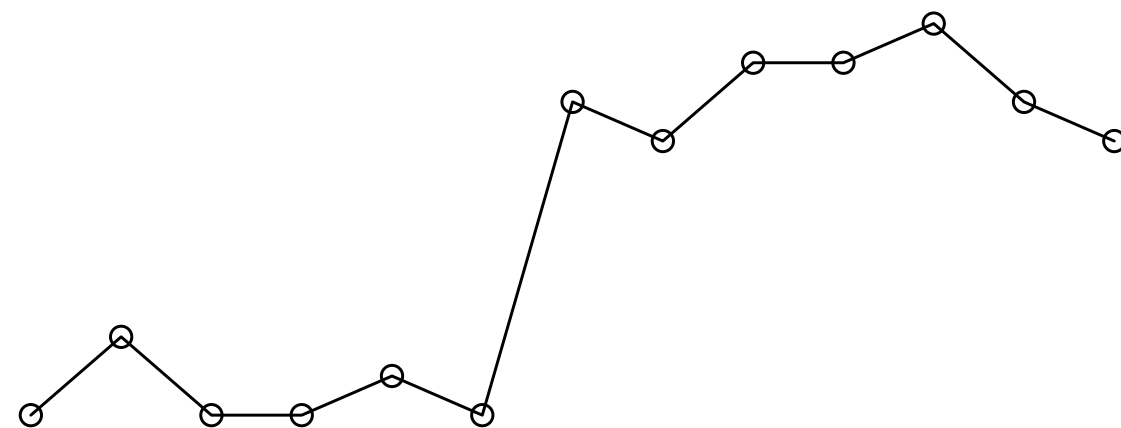
CGCGTCTG: 10

GCGTCTGG: 10

CGTCTGGC: 11

GTCTGGCC: 9

TCTGGCCT: 8



GCGTATTAC**A**CGTCTGGCCT

GCGTATTA: 8

CGTATTAC: 8

GTATTAC**A**: 1

TATTAC**A**C: 1

ATTAC**A**CG: 1

TTAC**A**CGT: 1

TAC**A**CGTC: 1

AC**A**CGTCT: 2

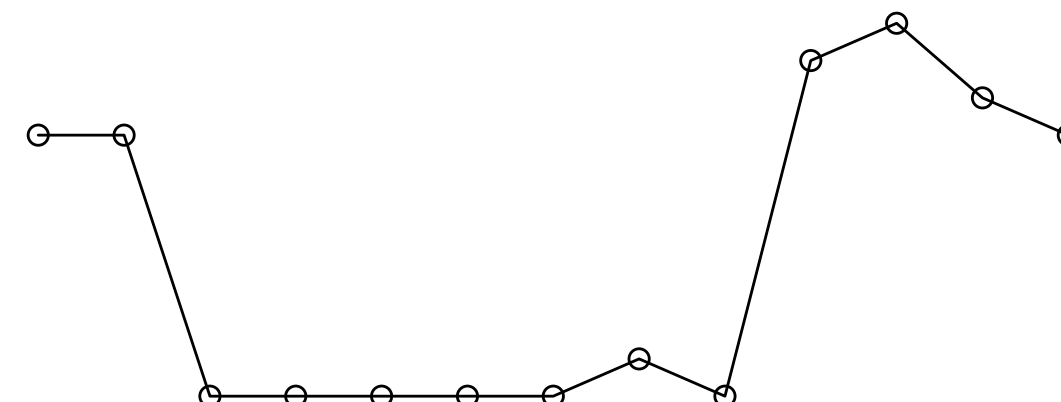
**C**ACGTCTG: 1

GCGTCTGG: 10

CGTCTGGC: 11

GTCTGGCC: 9

TCTGGCCT: 8



GCGTATTACGCGTCTGG**T**CT

GCGTATTA: 8

CGTATTAC: 8

GTATTACG: 9

TATTACGC: 9

ATTACGCG: 9

TTACGCGT: 12

TACGCGTC: 9

ACGCGTCT: 8

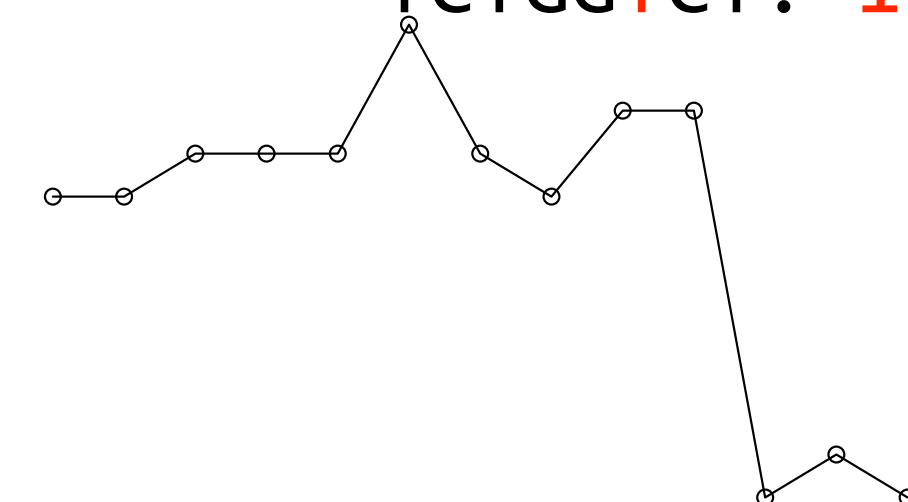
CGCGTCTG: 10

GCGTCTGG: 10

CGTCTGG**T**: 1

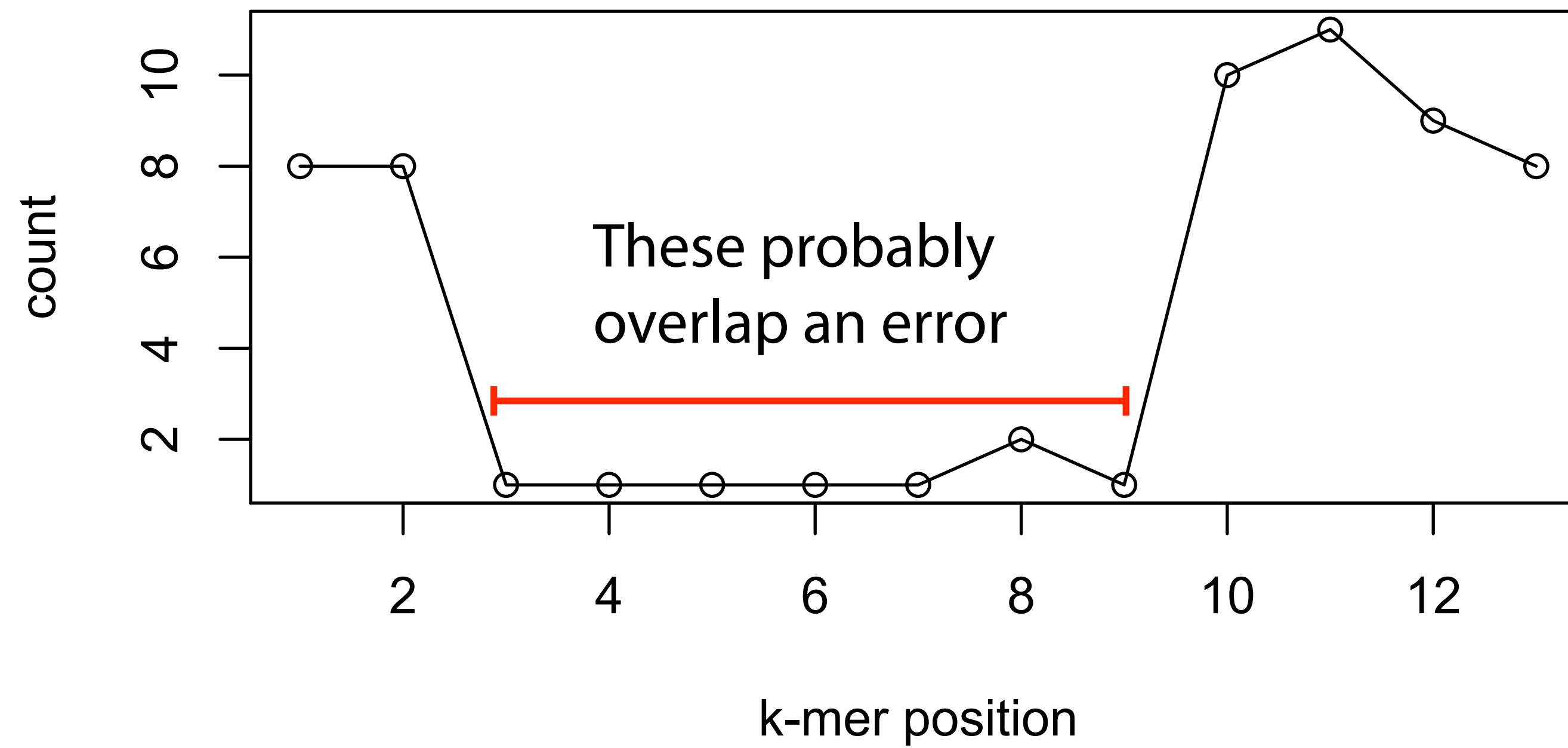
GTCTGG**T**C: 2

TCTGG**T**CT: 1



# Error correction

$k$ -mer count profile indicates where errors are



# Error correction

Simple algorithm: given a count threshold  $t$ :

For each read:

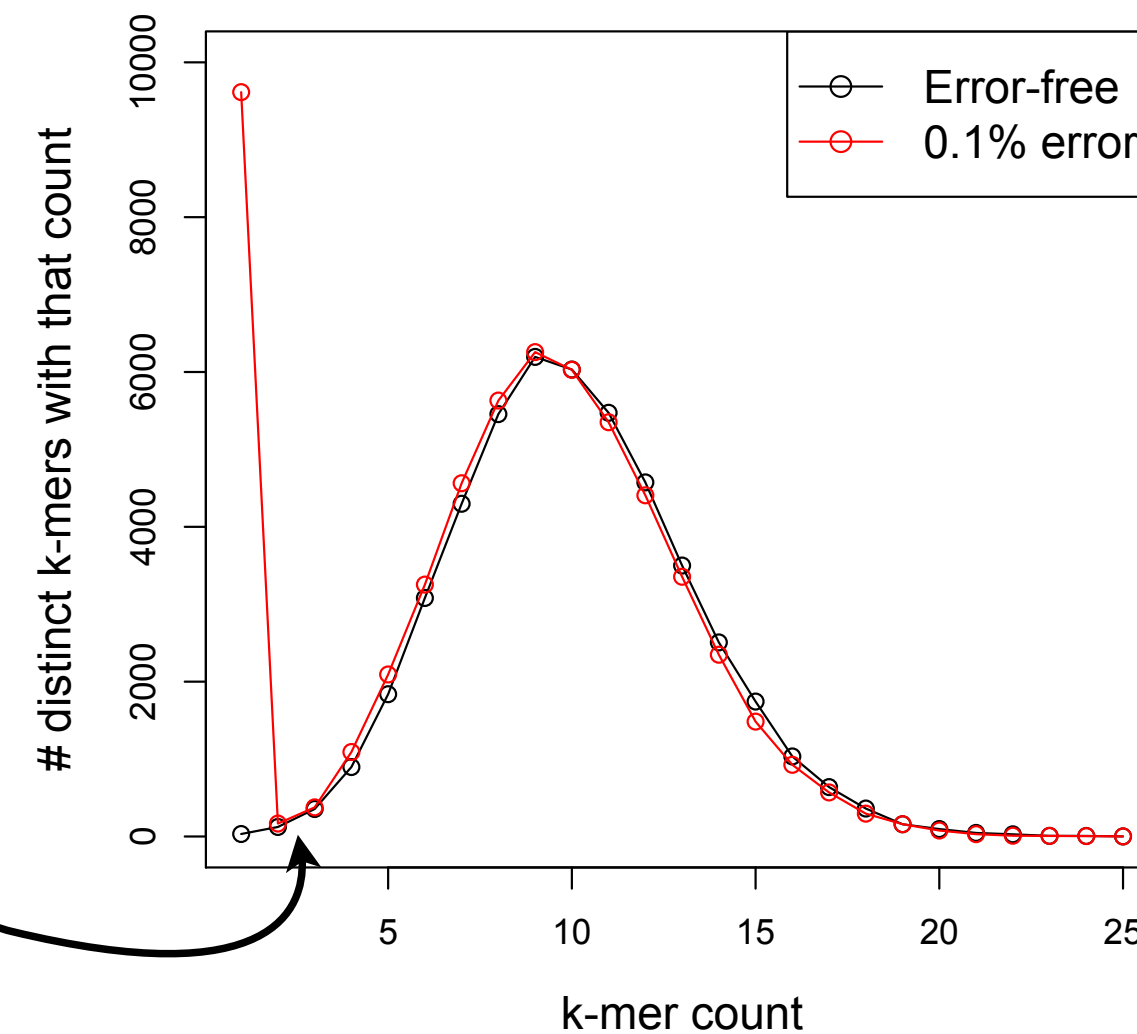
For each  $k$ -mer:

If  $k$ -mer count  $< t$ :

Examine  $k$ -mer's neighbors within certain Hamming/edit distance.

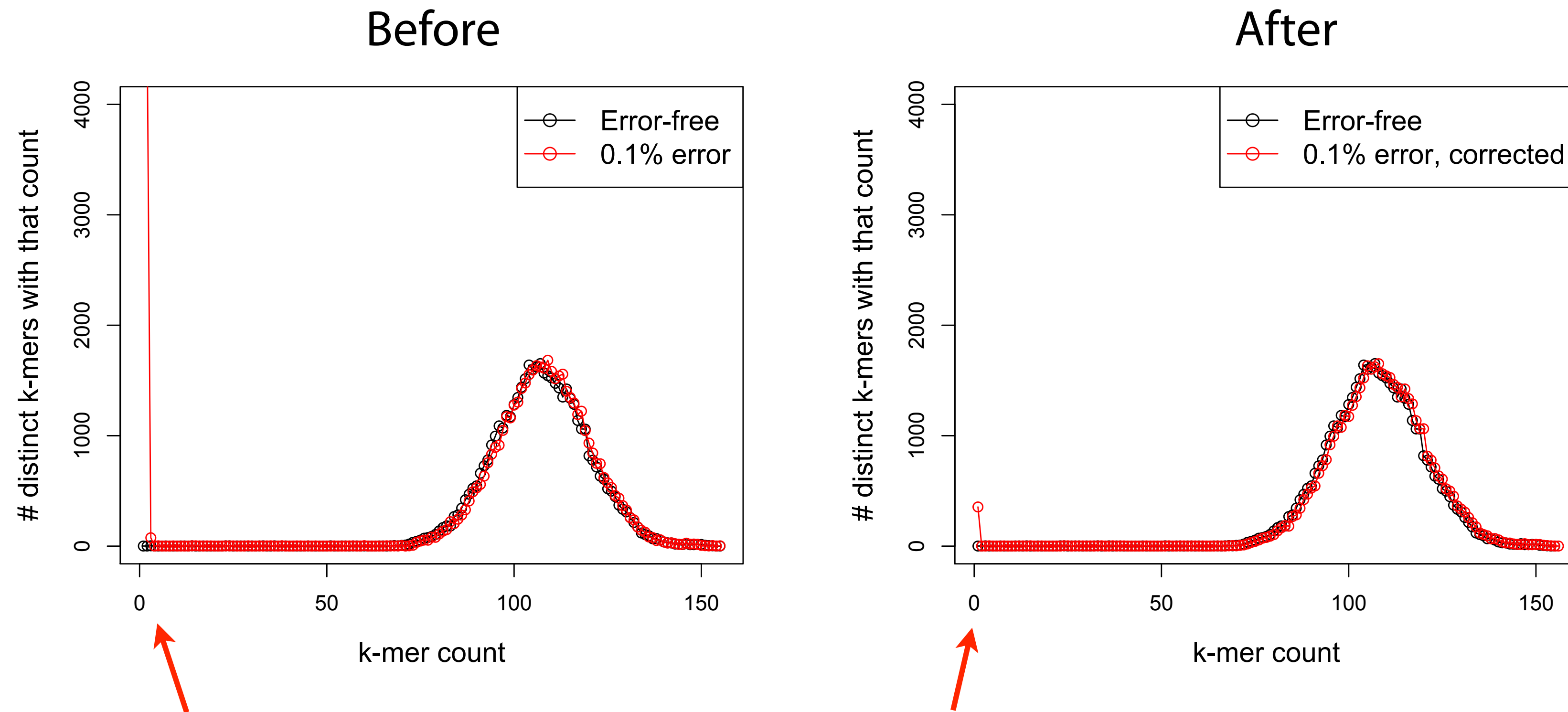
If neighbor has count  $\geq t$ , replace old  $k$ -mer with neighbor.

Pick a  $t$  that lies in the trough  
(the dip) between the peaks



# Error correction: results

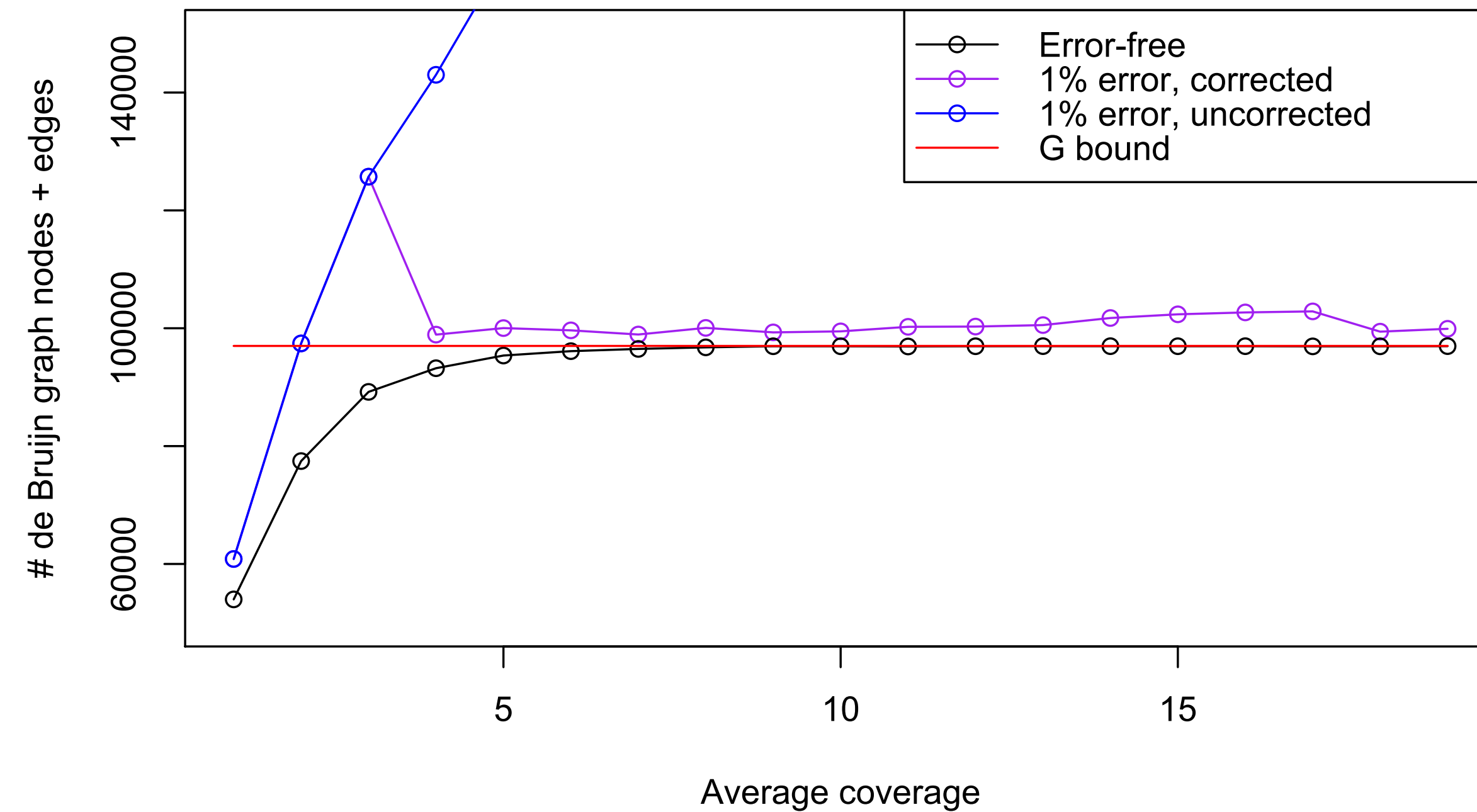
Corrects 99.2% of the errors in the example 0.1% error dataset



# Error correction: results

For **uncorrected** reads, De Bruijn graph size is off the chart

For **corrected** reads, De Bruijn graph size is near **G bound**



# Error correction

For error correction to work well:

Average coverage should be high enough and  $k$  should be set so we can distinguish infrequent from frequent  $k$ -mers

$k$ -mer neighborhood we explore must be broad enough to find frequent neighbors. Depends on error rate and  $k$ .

Data structure for storing  $k$ -mer counts should be substantially smaller than the De Bruijn graph

Otherwise there's no point doing error correction separately

Counts don't have to be 100% accurate; just have to distinguish frequent and infrequent